MATH 104 – Sample problems for first exam - Fall 2011

1. $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx =$

   (a) 0 (b) 2 (c) 4 (d) e^2 (e) $e^4 - e$

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2. We know that the graph of a function $\varphi(x)$ passes through the point (4,6). We also know that $-1 \le \varphi'(x) \le 3$ for all x in the interval $[0, 10]$. What are the minimum and maximum values possible values for $\varphi(7)$?

   (a) min = 7, max = 16 (b) min = 4, max = 7 (c) min = -1, max = 4
   (d) min = 6, max = 10 (e) min = 4, max = 13 (f) min = 3, max = 15

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3. $\displaystyle\int_1^e x(\ln x)\,dx =$

   (a) $\dfrac{e^2}{4} - \dfrac{4}{e^2}$ (b) $\dfrac{e^2}{2} - \dfrac{2}{e^2}$ (c) $\dfrac{3e^2}{4}$

   (d) $\dfrac{e^2}{4} + \dfrac{1}{4}$ (e) $\dfrac{3e^2}{2} - \dfrac{1}{4e^2}$ (f) $\dfrac{e^2}{4}$

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4. What is the volume of the solid obtained by rotating the part of the graph of $y = \cos x\sqrt{\sin x}$ between $x = 0$ and $x = \frac{\pi}{2}$ around the x-axis?

   (a) π (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{3}$

   (d) $\dfrac{\pi}{4}$ (e) $\dfrac{\pi}{6}$

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5. $\displaystyle\lim_{b\to\infty} \int_0^b \frac{\arctan x}{1+x^2}\,dx =$

   (a) $\dfrac{\pi^2}{2}$ (b) π^2 (c) $\dfrac{\pi^2}{4}$ (d) $\dfrac{\pi^2}{8}$ (e) $+\infty$

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6. The area between a curve $y = f(x)$ and the x-axis between $x = 1$ and $x = b$ is rotated around the x axis and the volume of the resulting solid is $\dfrac{\pi}{3}(\ln b)^3$, for all $b > 1$. What is $f(x)$?

   (a) $(\ln x)^2$ (b) $\dfrac{\ln x}{\sqrt{x}}$ (c) $\dfrac{\ln x}{x}$ (d) $\dfrac{(\ln x)^2}{x}$ (e) $\dfrac{(\ln x)^2}{\sqrt{x}}$

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7. $\int_0^{\pi} \sin^2(2x)\,dx =$

(a) 0 (b) 1 (c) 1/2 (d) $\pi/2$ (e) π (f) 2π

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8. Suppose that $f(x)$ and $g(x)$ are two differentiable functions, and we know the following values:

$$f(0) = 2, \quad f(1) = 3, \quad g(0) = 1, \quad g(1) = 4.$$

Then $\int_0^1 \Big(f(x)^2 g'(x) + 2f(x)f'(x)g(x)\Big)\,dx =$

(a) 6 (b) 12 (c) 14 (d) 16 (e) 32 (f) 48

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9. The area bounded by the curves $y = x^2 - 1$ and $y = 2x + 7$ is

(a) 4 (b) 9 (c) 16 (d) 24 (e) 30 (f) 36

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10. If $f(x)$ is continuous for all x and $\lim_{x\to 0} \dfrac{f(x)}{x} = 4$, then

(a) $f(0) = 1$ and $f'(0) = 4$ (b) $f(0) = 0$ and $f'(0) = 4$
(c) $f(0) = 4$ and $f'(0) = 0$ (d) $f(0) = 0$ and $f'(0) = 1$
(e) $f(0) = 0$ and $f'(0) = 0$ (f) $f(0) = 4$ and $f'(0) = 1$

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11. $\dfrac{d}{dx}\int_0^{\sin x} e^{t^2}\,dt =$

(a) $e^{\sin^2 x}$ (b) $\cos x\, e^{\sin^2 x}$ (c) $2\sin x\, \cos x\, e^{\sin^2 x}$
(d) $\cos x + e^{\sin^2 x}$ (e) $2\sin x\, e^{\sin^2 x} + e^{\sin^2 x}$ (f) $e^{\cos^2 x}$

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12. $\int_{1/e}^{e} \dfrac{(\ln x)^2}{x}\,dx =$

(a) $\dfrac{e^2}{2} - \dfrac{2}{e^2}$ (b) $\dfrac{e}{2} - \dfrac{2}{e}$ (c) $\dfrac{2e}{3}$

(d) $\dfrac{2}{3}$ (e) $\dfrac{e^2}{2} - \dfrac{1}{2e^2}$ (f) $\dfrac{2e^2}{3}$

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13. What is the average value of the function $f(x) = \sqrt{\dfrac{\arcsin x}{1 - x^2}}$ on the interval $[0, \frac{1}{2}]$?

(a) $\dfrac{\pi\sqrt{6}}{15}$ (b) $\dfrac{\pi^{3/2}\sqrt{2}}{54}$ (c) $\dfrac{\pi\sqrt{3}}{18}$ (d) $\dfrac{\pi^{3/2}\sqrt{6}}{27}$ (e) $\dfrac{\pi\sqrt{2}}{48}$

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14. What is the volume of the solid obtained by rotating the region bounded by the graphs of $y = \sqrt{x}$, $y = 2 - x$ and $y = 0$ around the x-axis?

(a) $\frac{5\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{2\pi}{5}$ (d) $\frac{11\pi}{6}$ (e) $\frac{5\pi}{3}$ (f) $\frac{7\pi}{6}$

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15. The functions $\cosh x$ and $\sinh x$ are defined as $\cosh x = \frac{1}{2}(e^x + e^{-x})$ and $\sinh x = \frac{1}{2}(e^x - e^{-x})$. Calculate the area bounded by the curves $y = \cosh x$, $y = \sinh x$, $x = 0$ and $x = b$. What is the limit of this area as $b \to \infty$?

(a) 1 (b) e (c) $e + \frac{1}{e}$ (d) $\frac{1}{e}$ (e) ∞

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16. Suppose $f(x)$ is a monotonically increasing function on the interval $[0, \infty)$, and that $f(0) = 2$ and $\lim_{x\to\infty} f(x) = 6$. Suppose $A(b)$ is the average value of $f(x)$ on the interval $[0, b]$ for $b > 0$. What is $\lim_{b\to\infty} A(b)$?

(a) 2 (b) 4 (c) 5
(d) 6 (e) 0 (f) ∞

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17. What is the volume obtained by revolving the region bounded by $y = x^2 - 4$ and $y = 4 - x^2$ around the line $x = 2$?

(a) $\frac{16\pi}{3}$ (b) $\frac{64\pi}{3}$ (c) $\frac{256\pi}{3}$ (d) $\frac{512\pi}{3}$ (e) $\frac{1024\pi}{3}$

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18. $\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx =$

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2 (e) $2\sqrt{2}$ (f) 3

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19. Compute $\int (\sin^2 x + 8)^7 \sin x \cos x\,dx$.

(a) $\frac{1}{16}(\sin^2 x + 8)^8 + C$ (b) $(\cos^2 x + 8)^7 + C$ (c) $\frac{1}{2}(\sin^2 x + 8)^8 + C$

(d) $\frac{1}{8}(\sin^2(x + 8)^8 + C$ (e) $\frac{1}{8}(\cos^2 x + 8)^8 + C$ (f) $\frac{1}{32}(\sin^2 x + 8)^7 + C$

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20. Suppose f is a continuous function and $\int_1^9 f(x)\,dx = 6$. Then $\int_1^3 xf(x^2)\,dx =$

(a) 6 (b) 4 (c) 3 (d) 2 (e) 1 (f) 0

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21. $\displaystyle\int_0^1 \frac{e^{\arctan(x)}}{1+x^2}\,dx =$

(a) $e^{\pi/4}-1$ (b) $e^{\pi/4}$ (c) $e-1$ (d) $\dfrac{\pi}{4}$ (e) $\dfrac{\pi}{4}-e$ (f) $e^{\pi/2}-e^{\pi/4}$

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22. Find the area between the graphs of $y=\sin x$ and $y=\cos x$ for $0\le x\le \pi/4$

(a) $\dfrac{\sqrt{2}}{2}$ (b) $\sqrt{2}$ (c) $\sqrt{2}-1$ (d) $\dfrac{\sqrt{2}}{2}+1$ (e) $\dfrac{\sqrt{2}}{2}-1$ (f) 1

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23. $\displaystyle\int_0^{\pi/3} \sec^3 x\,\tan x\,dx$

(a) $\dfrac{\sqrt{2}}{6}-\dfrac{1}{6}$ (b) $\dfrac{\sqrt{3}}{6}-1$ (c) $\dfrac{11}{3}$ (d) $\dfrac{7}{3}$ (e) $\dfrac{\sqrt{2}}{3}-\dfrac{1}{3}$ (f) $\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}$

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24. The base of a solid is the triangle in the xy-plane with vertices (0,0), (1,0) and (0,1). Cross-sections of the solid perpendicular to the x-axis are squares. What is the volume of the solid?

(a) $\dfrac{2}{3}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{3}{4}$ (d) $\dfrac{4}{3}$ (e) $\dfrac{5}{4}$ (f) $\dfrac{1}{3}$

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25. Find the volume of the solid obtained by rotating the area between the graphs of $y=x^2$ and $x=2y$ around the y-axis.

(a) $\dfrac{2\pi}{45}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{96}$ (d) $\dfrac{\pi}{24}$ (e) $\dfrac{\pi}{180}$ (f) $\dfrac{3\pi}{64}$

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26. $\displaystyle\int_1^2 \frac{\ln x}{x^2}\,dx =$

(a) $4-\ln(\sqrt{2})$ (b) $\dfrac{1-\ln(2)}{2}$ (c) $\ln(2)-1$ (d) $\ln(2)$ (e) $1-\dfrac{\ln(2)}{2}$ (f) $\dfrac{\ln(2)}{2}$

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27. Find the average value of the function $f(x)=t\sqrt{16+t^2}$ on the interval $0\le x\le 3$.

(a) $\sqrt{2}$ (b) $\dfrac{2\sqrt{2}}{9}$ (c) $\dfrac{32}{9}$ (d) $\dfrac{61}{3}$ (e) $\dfrac{\sqrt{2}}{3}$ (f) $\dfrac{61}{9}$

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28. Find the volume obtained by rotating the region between the x-axis, the y-axis and the line $x+y=1$ around the line $x=-2$.

(a) $\dfrac{7\pi}{3}$ (b) $\dfrac{7\pi}{6}$ (c) $\dfrac{19\pi}{3}$ (d) $\dfrac{19\pi}{6}$ (e) $\dfrac{19\pi}{12}$ (f) $\dfrac{7\pi}{12}$

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29. Find the volume of the solid obtained by rotating the region between the graph of $y = \sin(x/2)$ and the x axis for $0 \le x \le 2\pi$ around the y-axis.

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) π^2 (d) $2\pi^2$ (e) $4\pi^2$ (f) $8\pi^2$

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30. Find the volume of the solid obtained by rotating the region between the graph of $y = \sin(x/2)$ and the x axis for $0 \le x \le 2\pi$ around the x-axis.

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{4}$ (c) $\frac{\pi^2}{2}$ (d) π^2 (e) $2\pi^2$ (f) $4\pi^2$

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1. Find the area between the graphs of $y = 1$ and $y = x^4$.

(a) $\frac{8}{7}$ (b) $\frac{16}{9}$ (c) $\frac{16}{5}$ (d) $\frac{12}{7}$ (e) $\frac{8}{5}$ (f) 1

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2. Find the volume of the solid obtained by rotating the area between the graphs of $y = x\sqrt{2-x}$ and $y = 0$ around the x-axis.

(a) $\frac{\pi}{30}$ (b) $\frac{\pi}{24}$ (c) $\frac{4\pi}{3}$ (d) $\frac{\pi}{12}$ (e) $\frac{3\pi}{64}$ (f) $\frac{\pi}{180}$

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3. Find the volume of the solid obtained by rotating the area between the graph of $y = x\cos x$ and the x-axis for $0 < x < \frac{\pi}{2}$ around the y-axis.

(a) $\frac{\pi^3}{2}$ (b) $\frac{2\pi^3}{3}$ (c) $\frac{\pi^3}{2} - 8\pi$ (d) $2\pi^3 - 8\pi$ (e) $2\pi^3 - 4\pi$ (f) $\frac{\pi^3}{2} - 4\pi$

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4. Find the volume obtained by rotating the square with corners at the points $(0,0)$, $(0,1)$, $(1,1)$ and $(1,0)$ around the line $x = 4$.

(a) 3π (b) 5π (c) 7π (d) 9π (e) 11π (f) 13π

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5. Let $V(b)$ be the volume obtained by rotating the area between the x-axis and the graph of $y = \frac{1}{x^3}$ from $x = 1$ to $x = b$ around the x-axis. What is $\lim_{b\to\infty} V(b)$?

(a) $\frac{\pi}{5}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$ (e) π (f) ∞

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6. Let $V(a)$ be the volume obtained by rotating the area between the x-axis and the graph of $y = \frac{1}{x^{3/2}}$ from $x = a$ to $x = 1$ around the y-axis. What is $\lim_{a\to 0+} V(a)$?

(a) $\frac{\pi}{5}$ (b) $\frac{4\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) 4π (e) $\frac{6\pi}{5}$ (f) ∞

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7. $\int_0^1 \arcsin x \, dx$

(a) $\frac{\pi}{3} - \frac{1}{3}$ (b) $\frac{\pi}{6} - \ln 2$ (c) $\frac{\pi}{2} - 1$ (d) $\frac{\pi}{4} - \frac{\ln 2}{2}$ (e) $\pi - \frac{\ln 2}{6}$ (f) $\frac{\pi}{4} - \frac{1}{2}$

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8. $\int_0^{\sqrt{\pi}} x \sin^2(x^2) \, dx$

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$ (e) $\frac{\pi}{4}$ (f) $\frac{3\pi}{4}$

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9. Find the area between the graphs of $y = x^4 + 4x^2$ and $y = 4x^3$.

(a) $\frac{1}{30}$ (b) $\frac{8}{15}$ (c) $\frac{16}{15}$ (d) $\frac{27}{10}$ (e) $\frac{81}{10}$ (f) $\frac{512}{15}$

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10. The average value of the function $f(x)$ on the interval $[0, b]$ is $\sqrt{b}$. What is $f(x)$?

(a) $(1+x)e^x$ (b) $\frac{e^x}{1+x}$ (c) $\cos x + x \sin x$ (d) $\sin x + x \cos x$ (e) $\frac{3}{2\sqrt{x}}$ (f) $\frac{3\sqrt{x}}{2}$

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31. Calculate the length:

(a) of the part of $y = \frac{1}{3}(x^2+2)^{3/2}$ from $x = 0$ to $x = 1$. (Answer: 4/3)

(b) of the part of $y = x^{2/3}$ from $x = -1$ to $x = 8$ (careful!) (Answer: $\frac{80\sqrt{10} + 13\sqrt{13} - 16}{27}$)

(c) of the part of $y = \ln(\cos x)$ for $0 \le x \le \pi/4$. (Answer: $\ln(\sqrt{2}+1)$)

(d) of the part of $y = \ln x$ for $1 \le x \le 2$
(Answer: $\sqrt{5} - \sqrt{2} + \frac{1}{2}\ln(\sqrt{5}-1) - \frac{1}{2}\ln(\sqrt{2}-1) - \frac{1}{2}\ln(\sqrt{5}+1) + \frac{1}{2}\ln(\sqrt{2}+1)$)

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32. Calculate the surface area obtained by rotating:

(a) $y = \sqrt{x}$ around the x axis for $0 \le x \le 4$ (Answer: $\frac{\pi}{6}(17\sqrt{17} - 1)$)

(b) $y = x^3$ around the x axis for $1 \le x \le 2$ (Answer: $\frac{\pi}{27}(145\sqrt{145} - 10\sqrt{10})$)

(c) $y = \sqrt{9 - x^2}$ around the y axis for $1 \le x \le 3$. (Answer: $12\pi\sqrt{2}$)

(d) $y = x^2$ around the x axis for $0 \le x \le 1$. (Answer: $\frac{\pi}{32}(18\sqrt{5} - \ln(\sqrt{5}+2))$)