MATH 371 – Homework 9– November 9, 2013

1. Factor $x^3 + x + 1$ into irreducible factors in $\mathbb{F}_p[x]$ for $p = 2, 3, 5$.

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2. (a) Show that $x^5 - x + 1$ is an irreducible polynomial in $\mathbb{F}_3[x]$.

(b) Conclude that $E = \mathbb{F}_3[x]/\langle x^5 - x + 1\rangle$ is a field. How many elements does it have?

(c) Let $\alpha = [x]$ in E. What is the multiplicative inverse of α in E?

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3. How many monic irreducible polynomials are there of degree 12 in $\mathbb{F}_7[x]$? Find one.

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4. (a) Calculate the value of the Legendre symbol $\left(\frac{713}{1009}\right)$.

(b) Show that if p is an odd prime number then

$$\left(\frac{3}{p}\right) = \begin{cases} 1 & \text{if} \quad p \equiv \pm 1 \pmod{12} \\ -1 & \text{if} \quad p \equiv \pm 5 \pmod{12} \end{cases}$$

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5. (a) Calculate the remainder on division of the polynomial $x^7y^2 + x^3y^2 - y + 1$ by $(xy^2 - x\,,\ x = y^3)$. Use the lexicographic order.

(b) Repeat part (a) using the graded lexicographic order.

(c) Repeat parts (a) and (b) with the two polynomial divisors reversed.

(d) Suppose $I \subset k[x_1, \ldots, x_n]$ is a principal ideal, that is $I = \langle f\rangle$. Show that any finite subset of I containing λf for some non-zero $\lambda \in k$ is a Gröbner basis for I.