MATH 371 – Homework 10– November 21, 2013

1. Along the lines of what we did in class, express $x^4 + y^4$ as a function of $t_1 = x + y$ and $t_2 = xy$.

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2. Use Lagrange multipliers to find the max and min of $x^3 + 2xyz - z^2$ subject to the constraint $x^2 + y^2 + z^2 = 1$. (Let λ be the Lagrange multiplier. To solve the four equations in four unknowns that result, calculate a Gröbner basis using lex order with $\lambda > x > y > z$ so you get rid of λ first.)

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3. Suppose you have numbers x, y, and z that satisfy the equations

$$\begin{aligned} x + y + z &= 3 \\ x^2 + y^2 + z^2 &= 5 \\ x^3 + y^3 + z^3 &= 7 \end{aligned}$$

(a) Prove that $x^4+y^4+z^4 = 9$. (Hint: Find a [reduced] Gröbner basis for the ideal corresponding to the above equations, and then find the remainder of $x^4 + y^4 + z^4$ with respect to that basis)

(b) What are $x^5 + y^5 + z^5$ and $x^6 + y^6 + z^6$?