Math 410 **Dr. DeTurck**
Final Exam **Due Thursday, December 17, 2009**

As we discussed in class, you should work in groups of approximately six. Each group should hand in one set of solutions. You will be assigned a 20-minute time slot between 9 and 11 am on December 17 for oral presentations.

1. Evaluate $\displaystyle\int_0^\infty \frac{x^2}{2x^4+5x^2+2}\,dx$.

2. Show that if $|\lambda|<1$,
$$\int_0^\infty \frac{x^\lambda}{x^2+3x+2}\,dx = \frac{\pi}{\sin\lambda\pi}(2^\lambda-1).$$

3. Find the harmonic function $\varphi(x,y)$ on the upper half-plane ($y>0$) that satisfies $\varphi(x,0)=0$ if $x>2$ or if $x<-2$, and $\varphi(x,0)=1$ if $-2<x<2$.

4. How many solutions of $e^z = 5z^3-1$ are inside the unit disk $|z|\le 1$?

5. Show that
$$\Gamma(5z) = \frac{3125^z\sqrt{5}}{20\pi^2}\Gamma(z)\Gamma(z+\frac{1}{5})\Gamma(z+\frac{2}{5})\Gamma(z+\frac{3}{5})\Gamma(z+\frac{4}{5}).$$

6. Find the infinite product formula for $\sinh\pi z$.