

Reading: Textbook, Chapter 2, pp 32-45, then preview 45-53.

Practice problems: (don't hand these in)

1. Calculate $\int_{\gamma} e^{\pi z} dz$ where γ is any curve that starts at $z = i$ and ends at $z = i/2$.
2. Ditto for $\int_{\gamma} \cos \frac{z}{2} dz$ except γ is any curve that starts at $z = 0$ and ends at $z = \pi + 2i$.
3. Ditto for $\int_{\gamma} \frac{dz}{z}$, where γ is any curve that starts at $z = -2i$ and ends at $z = 2i$, and that (except for the endpoints) is contained entirely in the *right* half-plane (where the real part of z is positive). Why do we need this assumption? What if we change it to *left* half-plane?

Problems to hand in:

1. Textbook page 64, problems 2, 3, 4.
2. Explain carefully why $\int_{-\infty}^{\infty} \frac{x^2 + 4 - 3i}{(x + 2i)^4} dx = 0$ (where x is real even if the integrand isn't).
3. Show that if $z_1 \neq 0$, $z_2 \neq 0$, then

$$\int_{\gamma} \frac{dz}{z^2} = \frac{1}{z_1} - \frac{1}{z_2},$$

where γ is any curve contained in a simply-connected domain which does not contain the origin. Then show how it follows that for *any* closed contour γ that does not pass through the origin,

$$\int_{\gamma} \frac{dz}{z^2} = 0.$$

4. Explain why $\int_0^{\infty} x^{\alpha-1} e^{-x} dx$ converges provided α is any *complex* number with positive real part. (This expression defines the Gamma function $\Gamma(\alpha)$ for these values of α .)