Math 410 **Dr. DeTurck**
Assignment 4 **Due Tuesday, October 20, 2009**

Reading: Textbook, Chapter 2, pp 32-45, then preview 45-53.

Practice problems: (don't hand these in)

1. Calculate $\int_\gamma e^{\pi z}\,dz$ where γ is any curve that starts at $z = i$ and ends at $z = i/2$.
2. Ditto for $\int_\gamma \cos\frac{z}{2}\,dz$ except γ is any curve that starts at $z = 0$ and ends at $z = \pi + 2i$.
3. Ditto for $\int_\gamma \frac{dz}{z}$, where γ is any curve that starts at $z = -2i$ and ends at $z = 2i$, and that (except for the endpoints) is contained entirely in the *right* half-plane (where the real part of z is positive). Why do we need this assumption? What if we change it to *left* half-plane?

Problems to hand in:

1. Textbook page 64, problems 2, 3, 4.
2. Explain carefully why $\int_{-\infty}^{\infty} \frac{x^2+4-3i}{(x+2i)^4}\,dx = 0$ (where x is real even if the integrand isn't).
3. Show that if $z_1 \neq 0$, $z_2 \neq 0$, then
$$\int_\gamma \frac{dz}{z^2} = \frac{1}{z_1} - \frac{1}{z_2},$$
where γ is any curve contained in a simply-connected domain which does not contain the origin. Then show how it follows that for *any* closed contour γ that does not pass through the origin,
$$\int_\gamma \frac{dz}{z^2} = 0.$$
4. Explain why $\int_0^\infty x^{\alpha-1}e^{-x}\,dx$ converges provided α is any *complex* number with positive real part. (This expression defines the Gamma function $\Gamma(\alpha)$ for these values of α.)