Math 410 **Dr. DeTurck**
Assignment 5 **Due Thursday, October 29, 2009**

Reading: Textbook, Chapter 2, pp 53-60, then look through 60-64, then read 71-83.

Practice problems: (don't hand these in)

1. Textbook page 65, problem 6, 10 (consider $\overline{z}$)
2. Textbook page 103, problems 1 and 2
3. Show that $\displaystyle\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\, dx = \frac{5\pi}{12}$.
4. Show that if $a > 0$, then $\displaystyle\int_{-\infty}^{\infty} \frac{x^6}{(a^4 + x^4)^2}\, dx = \frac{3\sqrt{2}\,\pi}{16a}$
5. Show that if a function $f(z)$ is holomorphic in the entire complex plane, and if there is a constant $M > 0$ so that $|f(z)| \le M|z|$ for all z, then f must be a linear function (use the idea in the proof of Liouville's theorem).

Problems to hand in:

1. Textbook page 65, problems 7, 8, 9, 11, 12
2. Textbook page 103, problames 3, 4, 5
3. Calculate $\displaystyle\int_0^{\infty} \frac{x^2}{x^4 + 6x^2 + 13}\, dx$ (the answer might be $\displaystyle\frac{\pi}{4}\left(\frac{3+\sqrt{13}}{2}\right)^{-1/2}$).
4. Calculate $\displaystyle\int_0^{\infty} \frac{x^2}{(x^2 + a^2)^3}\, dx$ for a real (the answer might be $\displaystyle\frac{\pi}{16a^3}$).
5. Show that if $f(z)$ is holomorphic in the entire plane, and if there is a constant $M > 0$ and a positive integer n so that $|f(z)| \le M|z|^n$, then f must be a polynomial.
6. Show that the sum of the series
$$\frac{z}{1-z} + \frac{z}{z^2-1} + \frac{z^2}{z^4-1} + \frac{z^4}{z^8-1} + \cdots$$
is 1 for $|z| < 1$, but is 0 for $|z| > 1$. Does this contradict the principle of analytic continuation?