

1. For which values of z is $z^2 = |z|^2$? For which values of z is $z^2 = i|z|^2$?
2. Let $f(z) = z + 1/z$. What is the image of the unit circle under the mapping defined by f ?
3. On the domain $\{z = x + iy, 0 \leq x \leq 2\pi, 0 \leq y \leq 2\pi\}$, what is the maximum value of $|\cos z|$?
4. Let $u(x, y) = 2x - xy$. Find a function $v(x, y)$ so that

$$f(x + iy) = u(x, y) + iv(x, y)$$

is a holomorphic function. Express $f(z)$ in terms of z alone.

5. Find all the solutions of $\sin z = \sqrt{3}$.
6. Calculate $\int_{\gamma} \bar{z} dz$, $\int_{\gamma} \frac{dz}{\bar{z}}$, where γ is the unit circle, traversed once in the counterclockwise direction.
7. Give an example of a (nontrivial) simple closed curve γ for which

$$\int_{\gamma} \frac{dz}{z^2 + z + 1} = 0$$

and another for which

$$\int_{\gamma} \frac{dz}{z^2 + z + 1} \neq 0.$$

What is the value of the second integral over your curve?

8. Calculate

$$\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + 1} dx$$

by applying the Cauchy Integral Formula to

$$\int_{\gamma} \frac{e^{iz}}{(z+i)(z-i)} dz$$

where γ is the “standard” semicircular contour of radius R and letting R go to infinity. Be sure to estimate what happens on the circle part carefully.