241 Homework 1 - Due Monday May 22 at the beginning of class.

Review Please re-read section 7.6 of the text. Then, do the following problems from the book:

a) *7.6.9* If it is a vector space, find the dimension and exhibit a basis.
b) *7.6.10* If it is a vector space, find the dimension and exhibit a basis.
c) *7.6.16* If it is a vector space, find the dimension and exhibit a basis.
d) *7.6.20*
e) *7.6.29*

For a) and b) use Definition 7.5 (it's long, but instructive to do once, unfortunately). For c) and d) use Theorem 7.4 if it applies. Notice that in general showing that something is a subspace of another vector space is a lot easier than checking it's a vector space directly.

Orthogonal Functions and Fourier Series:

a) *12.1.4*
b) *12.1.10*
c) *12.1.19* Show that even if you also include $\{\cos x, \cos 2x, \cos 3x, ...\}$ it's still not complete (who's missing?).
d) *12.2.2*
e) *12.2.4*
f) *12.3.14*
g) *12.3.38*
h) Let X be a set consisting of two points (say $x = 0$ and $x = 1$ in the real line if that makes you more comfortable). Identify the vector space $C(X)$ - what does a typical vector in this vector space look like? Find the dimension of the vector space and exhibit a basis. Does the fact that we're looking at continuous functions have any effect on the problem?
i) Show that the set $\{e^x, e^{2x}.e^{3x}\}$ is linearly independent in $C([0, 1])$. (Hint, try plugging in some specific points on $[0, 1]$ and reduce to a "typical" linear algebra problem.)
j) Find a vector orthogonal to e^x is $L^2(-1, 1)$. Find a vector orthogonal to e^x in $L^2(0, 1)$. (There's a lot of them - L^2 is hugely infinite dimensional, so just find any one you can....)