**241 Homework 3** - Due Tuesday June 13 at the beginning of class.

**Some Leftover PDE Questions:**

*a)* 13.1.11

*b)* 13.1.16

**Fourier Theory with Complex Numbers:**

*a)* Suppose $f(x)$ is a real-valued function with Fourier Series $f(x) = \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$ on $(-\pi, \pi)$. Find expressions for the $c_n$'s in terms of the $a_k$'s and $b_k$'s such that $f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$.

*b)* Conversely, suppose $f(x)$ is a real-valued function with Fourier Series $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$ on $(-\pi, \pi)$. Find expressions for the $a_k$'s and $b_k$'s in terms of the $c_n$'s such that $f(x) = \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$.

*c)* Suppose that $f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$ on $(-\pi, \pi)$. Give a (non-trivial) condition on the $c_k$'s such that $f(x)$ is a real-valued function. (*Hint:* $\overline{f(x)} = ?$)

**Basic Properties of Complex Numbers:**

*a)* 17.1.15. After this, write out a general expression for $\frac{a+ib}{c+id}$ in the form $x + iy$.
*b)* 17.1.27. Do the same gives $z = re^{i\theta}$.
*c)* 17.2.29.
*d)* 17.3.22
*e)* 17.4.23
*f)* 17.5.6
*g)* 17.5.28
*h)* 17.6.23. What is the Principal Value?
*i)* 17.6.38
*j)* 17.7.15. Do the same for $\sin z = -1$.