Name :

Midterm for Math114, Summer 2009

Write clearly, justify your answers, do not forget your name.
You will get no credit for guessing, but you will get partial credit for relevant work.

1. Assume $w = w(x, y, z)$, with $x = x(t)$, $y = y(s)$, $z = z(t, s)$. If we know that $x(1) = 3$, $y(2) = 2$, $z(1, 2) = 1$, $x'(1) = -1$, $y'(2) = 2$, $z_s(1, 2) = 3$, $w_x(3, 2, 1) = 1$, $w_y(3, 2, 1) = 2$, $w_z(3, 2, 1) = 3$,then $\frac{\partial w}{\partial s}$ at $(t, s) = (1, 2)$ is:

A: 0
B: 1
C: 7
$\boxed{\text{D}}$: 13
E: None of the above.

$$\frac{\partial w}{\partial s} = \cancel{\frac{\partial w}{\partial x} \cdot \overset{0}{\frac{\partial x}{\partial s}}} + \frac{\partial w}{\partial y} \cdot \frac{dy}{ds} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s}$$

$$= 0 + 2 \cdot 2 + 3 \cdot 3$$

$$\boxed{= 13}$$

2. Consider the plane tangent to the surface $z = x\sin(x-y)$ at the point $(2,2,0)$. This plane intersects the y-axis at the point where y=

A: -1

B: 1

$\boxed{\text{C:}}$ 0

D: 2

E: None of the above.

$$\frac{\partial z}{\partial x} = \sin(x-y) + x\cos(x-y)$$

$$\frac{\partial z}{\partial y} = -x\cos(x-y)$$

$$\Rightarrow \frac{\partial z}{\partial x}(2,2) = 0 + 2 = 2$$

$$\frac{\partial z}{\partial y}(2,2) = -2$$

$\Rightarrow$ Tangent plane given by

$$z = 2(x-2) - 2(y-2) + 0$$

$$= 2x - 2y$$

$z = x = 0 \Rightarrow y = 0$

3. If $f(x,y) = \frac{2}{\sqrt{x^2+y^2-4}}$, find the domain and range?

A. Domain = all (x,y) with $x^2+y^2<4$, Range = $[0,\infty)$
B. Domain = all (x,y) with $x^2+y^2>4$ and , Range = $[0,\infty)$
C. Domain = all (x,y) with $x^2+y^2>4$, Range = $(1,\infty)$
D. Domain = all (x,y) with $x^2+y^2<4$, Range = $(1,\infty)$
$\boxed{\text{E.}}$ None of the above.

Domain

Need $x^2+y^2-4>0$

$\Rightarrow x^2+y^2>4$ ✓

Range

Claim: Range = $(0,\infty)$

Check $a^2 = \frac{2}{\sqrt{x^2+y^2-4}} \Rightarrow \sqrt{x^2+y^2-4} = \frac{2}{a^2}$

$\Rightarrow x^2+y^2 = \frac{4}{a^4}+4$

Set $y=0 \Rightarrow$ $x = \sqrt{\frac{4}{a^2}+4}$

works ✓

4. Find the equation of a plane that is parallel to the plane $x-y+2z = 11$ and that contains the line $\langle 1+3t, 1+t, 2-t\rangle$:

A. $x - 2y + z = 1$

B. $x - y + 2z = 6$

C. $x - 2y + z = 2$

$\boxed{\text{D.}}$ $x - y + 2z = 4$

E. None of the above.

$\Rightarrow$ contains $(1,1,2)$, has normal $= (1,-1,2)$

$$(x-1) - (y-1) + 2(z-2) = 0$$

$$x - y + 2z = 4$$

5. If $f(x,y) = \frac{2x^2\sin(xy)}{3x^2+4y^2}$, then

$$\lim_{(x,y)\to(0,0)} f(x,y) =$$

$\boxed{\text{A.}}$ 0
B. 1
C. -1
D. $\frac{2}{3}$
E. The limit does not exist.

$$|f(x,y)| = \left|\frac{2x^2\sin(x\cdot y)}{3x^2+4y^2}\right| = |\sin(x\cdot y)| \cdot \overbrace{\left|\frac{2x^2}{3x^2+4y^2}\right|}^{\leq 1}$$

$$\leq |\sin(xy)|$$

$$\to 0 \quad \text{as} \quad (x,y)\to 0$$

6. The length of the curve $r(t) = \langle \sin^2(t), 2, \cos^2(t) \rangle$ where $0 \leq t \leq \frac{\pi}{4}$ is:

A. $\frac{1}{2}$

$\boxed{\text{B.}}$ $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. 1

E. None of the above.

$$r'(t) = (2\sin(t)\cos(t), 0, -2\cos(t)\sin(t))$$

$$\Rightarrow \ell = \int_0^{\pi/4} \sqrt{4\sin^2 t \cos^2 t + 4\cos^2 t \sin^2 t}\, dt$$

$$= \int_0^{\pi/4} 2\sqrt{2}\sin t \cos t\, dt$$

$$= \sqrt{2}\sin^2 t \Big|_0^{\pi/4} = \sqrt{2}\cdot\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{\sqrt{2}}{2}$$

7. Assume $\frac{\partial f}{\partial y} = xe^y + xye^y$. Which of the following could be $\frac{\partial f}{\partial x}$:

A. xe^y

B. ye^y

C. xye^y

D. $y + xe^y$

E. None of the above.

B

If $f(x,y) = xye^y$, then $\frac{\partial f}{\partial y} = xe^y + xye^y$.

In this case $\frac{\partial f}{\partial x} = ye^y$

8. Find the position vector of a particle that starts at the position $\langle -1, 1, 1 \rangle$, with the velocity $v(t) = \langle 3t^2, 4t^3, -3t^2 \rangle$

A. $r(t) = \langle t^3, t^4, -t^3 \rangle$

B. $r(t) = \langle t^3 + 1, t^4 - 1, -t^3 + 1 \rangle$

$\boxed{\text{C.}}$ $r(t) = \langle t^3 - 1, t^4 + 1, -t^3 + 1 \rangle$

D. $r(t) = \langle t^3 + 1, t^4 + 1, -t^3 - 1 \rangle$

E. None of the above.

$$r(t) = \int_0^t v(t)\, dt = (t^3, t^4, -t^3) + r_0$$

$$\Rightarrow \quad r(t) = (t^3 - 1, t^4 + 1, 1 - t^3)$$

9. Consider the vectors $\mathbf{a} =< 1, 1, 1 >$, $\mathbf{b} =< 1, -1, 0 >$, $\mathbf{c} =< 1, 1, 0 >$. Then $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$ is?

A. 0
B. 2
C. $< 0, 0, 0 >$
D. $< 2, 1, 3 >$
E. None of the above.

Can't take the cross product of a vector + a scalar.

10. Find the value of x, such that the vectors $< x, 1, 2 >$ and $< 1, 2x, x-5 >$ are orthogonal.

A. $x = 1$

B. $x = -2$

C. $x = -1$

$\boxed{\text{D}}$. $x = 2$

E. None of the above.

Want

$$0 = (x, 1, 2) \cdot (1, 2x, x-5)$$

$$= x + 2x + 2x - 10$$

$$= 5x - 10$$

$$\Rightarrow x = 2$$