

Name :

Quiz 3 for Math114, Summer 2009

Write clearly, justify your answers, do not forget your name.  
You will get no credit for guessing, but you will get partial credit for relevant work.

1. Evaluate  $\int_0^2 \int_{\frac{x}{2}}^1 e^{y^2} dy dx$ .  
A. 0    B.  $e$     **C.  $e - 1$**     D.  $e + 1$     E. None of the above.

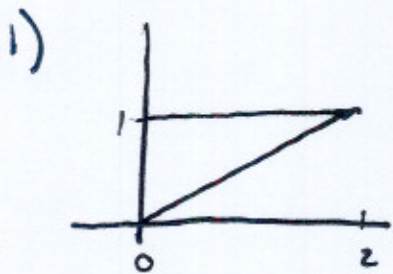
2. Find the absolute maximum value of  $f(x, y) = x^2 y^2$  on the set

$$D = \{(x, y) | x \geq 0, y \geq 0, x^2 + y^2 = 1\}$$

- A. 0    B.  $\frac{1}{2}$     C. 1    D. 2    **E. None of the above.**

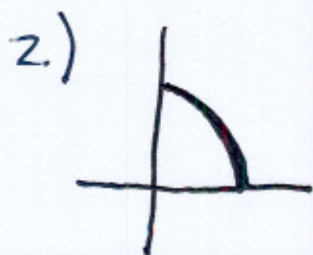
3. For which of the following points, the direction of fastest change of the function  $f(x, y) = x^2 - y^2$  is  $i + j$ :

- A. (1, 0)    B. (0, 1)    **C. (1, -1)**    D. (1, 1)    E. None of the above.



$$\int_0^2 \int_{x/2}^1 e^{y^2} dy dx = \int_0^1 \int_0^{2y} e^{y^2} dx dy$$

$$= \int_0^1 2y e^{y^2} dy = e^{y^2} \Big|_0^1 = \boxed{e - 1}$$



$$g = x^2 + y^2 \Rightarrow \nabla g = (2x, 2y), \quad \nabla f = (2xy^2, 2x^2y)$$
$$f = x^2 y^2$$

$$\Rightarrow \lambda 2x = 2xy^2 \Rightarrow x=0 \text{ or } \lambda = y^2$$

$$\lambda 2y = 2yx^2 \Rightarrow y=0 \text{ or } \lambda = x^2$$

$$\Rightarrow \text{set pts } (1, 0), (0, 1), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \Rightarrow \boxed{\text{max} = \frac{1}{4}}$$

$x=0 \Rightarrow f=0$      $y=0 \Rightarrow f=0$      $x^2=y^2 \Rightarrow f=\frac{1}{4}$

$$3) f = x^2 - y^2$$

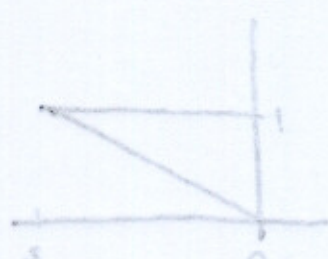
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$$\nabla f = (2x, -2y)$$

• Want  $\nabla f = k \cdot (1, 1) \Rightarrow$  want  $x > 0,$   
 $y = -x.$

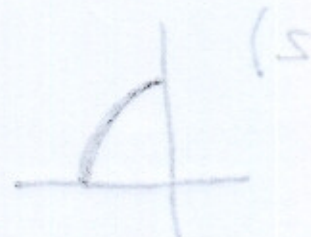
so  $(1, -1)$  works

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2x \\ -2y \end{pmatrix} \Rightarrow \begin{cases} 2x = 1 \\ -2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0.5 \\ y = 0 \end{cases}$$



$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2x \\ -2y \end{pmatrix} \Rightarrow \begin{cases} 2x = 1 \\ -2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0.5 \\ y = 0 \end{cases}$$

$$\begin{aligned} \nabla f = (2x, -2y) &= (0, 0) \Rightarrow \begin{cases} 2x = 0 \\ -2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \\ \nabla f = (2x, -2y) &= (1, 1) \Rightarrow \begin{cases} 2x = 1 \\ -2y = 1 \end{cases} \Rightarrow \begin{cases} x = 0.5 \\ y = -0.5 \end{cases} \end{aligned}$$



max/min at  $(0,0), (0.5, -0.5), (0.5, 0.5)$