

UNIVERSITY *of* PENNSYLVANIA  
 MATHEMATICS DEPARTMENT  
 MATHEMATICS 104/EXAMINATION 2  
 PROF. CROTTY

Name: \_\_\_\_\_ Penn ID #: \_\_\_\_\_

Section (check one):  002 (11 AM- noon)  004 (1-2 PM)

Your TA (check one):  Nukpezah  Russell  Yatauro  Yee

Instructions:

1. DO NOT DETACH THIS SHEET FROM YOUR TEST PAPER.
2. NO CALCULATORS may be used on this examination.
3. Cell phones should be off and placed out of sight.
4. No credit will be given for an answer without supporting work.
5. Write only your answer for each question in the appropriate space below. Show all work in the space provided on your test sheet.

Multiple Choice (5 pts. each)		Free Response (10 pts. each)	Maple (5 pts.)
1	9	1. a) Check one:	1 (2 pts)
2	10	<input type="checkbox"/> Spock lives!	2(3pts)
3	11	<input type="checkbox"/> He's dead, Jim!	
4	12	b) <input type="checkbox"/> yes <input type="checkbox"/> no	
5	13	2	
6	14	a)	
7	15	b)	
8		c)	
	subtotal	subtotal	subtotal

Raw Score \_\_\_\_\_ + Part Credit \_\_\_\_\_ – Unsupported \_\_\_\_\_ = \_\_\_\_\_

**Part 1: Multiple Choice**

Work each problem in the space provided. Write the letter corresponding to your answer on your answer sheet. Note that an answer with no supporting work will receive little or no credit.

1. Let  $f(x) = \sqrt{x} \ln x$ . Find the interval on which  $f$  is concave upward.
- a)  $(0, \infty)$                       b)  $(1, \infty)$                       c)  $(0, 1)$                       d)  $(0, e^{-1})$   
 e)  $(e^{-1}, \infty)$                       f)  $(0, e^{-2})$                       g)  $(e^{-2}, \infty)$                       h)  $(e, \infty)$
2. Evaluate:  $\lim_{x \rightarrow 0^+} \frac{e^{x^2} - 1}{x}$
- a) 4                      b) 1/4                      c) -2                      d)  $\infty$   
 e) 2                      f) 0                      g) 1/2                      h) -1/2
3. Suppose  $g$  is the inverse of a one-to-one differentiable function  $f$  and let  $G(x) = \frac{1}{g(x)}$ . If  $f(3) = 2$  and  $f'(3) = \frac{1}{9}$ , find the value of  $G'(2)$ .
- a) 9                      b) -1                      c) 1/9                      d) -9  
 e) 1                      f) 0                      g) 6                      h) -1/9
4. Let  $f(x) = e^{\ln 2x}$ . Find the value of  $f'(\pi)$ .
- a)  $\pi$                       b) -2                      c) 0                      d)  $-2\pi$   
 e) -1                      f)  $2\pi$                       g) 2                      h) 1
5. Evaluate:  $\int_0^\pi x \cos x \, dx$
- a)  $\pi$                       b)  $2\pi$                       c) 2                      d) 0  
 e) -2                      f) 1                      g) 1/2                      h)  $\pi/2$
6. Find the length of the curve  $y = \ln(\cos x)$ ,  $0 \leq x \leq \pi/4$ .
- a)  $\sqrt{2}$                       b)  $\sqrt{2} - 1$                       c)  $\ln(\sqrt{2}) - (1/2)$                       d)  $\ln(1 + \sqrt{2})$   
 e)  $\ln(1/\sqrt{2})$                       f)  $(\sqrt{2} - 1)/2$                       g)  $\ln(\sqrt{2}) - 1$                       h)  $1 - (1/\sqrt{2})$

7. Let  $d(t) = 100 ( 1 - e^{-0.1t} )$  be the number of cases of a certain disease at Podunk U.  $t$  days after the first case is reported. Which of the following best describes the spread of the disease?
- a) On the first day, there are 100 cases after which the number decreases daily to 0.
  - b) Initially, the disease spreads quickly, but then the rate of increase slows so that the total number of cases on any given day never exceeds 100.
  - c) The disease spreads quickly until the number of cases reaches 100 then the number of cases decreases gradually.
  - d) The number of cases remains essentially constant at about 100 on any given day.
  - e) The number of cases increases without limit according to an exponential growth law.
  - f) none of these

8. Evaluate:  $\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$
- |         |         |          |              |
|---------|---------|----------|--------------|
| a) 2/15 | b) 4/15 | c) 2/5   | d) 8/15      |
| e) 2/3  | f) 4/5  | g) 14/15 | h) divergent |

9. A function,  $F$ , is defined by the integral  $F = \int_0^{\sin 2x} \sin^{-1} t \, dt$ . Find  $F'(\frac{\pi}{6})$
- |                         |            |          |             |
|-------------------------|------------|----------|-------------|
| a) $\frac{\sqrt{3}}{2}$ | b) 2       | c) 0     | d) $\pi/6$  |
| e) $2\pi/3$             | f) $\pi/3$ | g) $\pi$ | h) $\infty$ |

10. Evaluate:  $\int_0^2 \frac{x^3}{\sqrt{x^2+4}} \, dx$
- |                                |                               |                               |                                |
|--------------------------------|-------------------------------|-------------------------------|--------------------------------|
| a) $\frac{8}{3}(2-\sqrt{2})$   | b) $\frac{10}{3}(2+\sqrt{2})$ | c) $4(3-2\sqrt{2})$           | d) $\frac{14}{3}(2\sqrt{2}-1)$ |
| e) $\frac{16}{3}(5-2\sqrt{2})$ | f) $6(4-\sqrt{2})$            | g) $\frac{20}{3}(4+\sqrt{2})$ | h) $12(5+\sqrt{2})$            |

11. Evaluate:  $\int_1^2 \frac{1}{x^3+x} \, dx$
- |                                |                                |                              |                                |
|--------------------------------|--------------------------------|------------------------------|--------------------------------|
| a) $3 \ln 2 - \ln 3$           | b) $\ln 2 - 2 \ln 3$           | c) $\ln 2 - \ln 5$           | d) $3 \ln 2 - \ln 5$           |
| e) $\frac{3 \ln 2 - \ln 3}{2}$ | f) $\frac{\ln 2 - 2 \ln 3}{2}$ | g) $\frac{\ln 2 - \ln 5}{2}$ | h) $\frac{3 \ln 2 - \ln 5}{2}$ |

12. Evaluate:  $\int_0^1 \frac{1}{\sqrt{4-x^2}} \, dx$
- |            |                         |                        |            |
|------------|-------------------------|------------------------|------------|
| a) $\pi/8$ | b) $-\frac{1}{2} \ln 2$ | c) $\frac{1}{2} \ln 2$ | d) $\pi/6$ |
| e) $\pi/4$ | f) $-1$                 | g) $1$                 | h) $\pi/2$ |

13. Evaluate:  $\int_0^e \frac{1}{x-1} dx$
- a)  $\ln(e-1)$                       b)  $e$                                       c) 0                                      d) 1  
 e)  $-\ln(e-1)$                       f)  $e^{-1}$                                       g)  $e^2$                                       h) divergent
14. Let  $f(x) = x^{\frac{1}{x}}$ . Find the value of  $f'(e)$ .
- a) 0                                      b) 1                                      c) 2                                      d) 3  
 e)  $1/e$                                       f)  $e$                                       g)  $e^{\frac{1}{e}}$                                       h) does not exist
15. Evaluate:  $\int_1^{\infty} \frac{\ln x}{x^2} dx$
- a) 0                                      b)  $1/4$                                       c)  $1/3$                                       d)  $1/2$   
 e) 1                                      f) 2                                      g) 3                                      h) divergent

## Part 2: Free Response

1. a) Mr. Spock, of Star Trek fame, has accidentally injected himself with the potentially lethal drug Cordrazine. He quickly calculates that the concentration  $y(t)$ , in parts per million (ppm), of the drug in his blood stream is given by:

$$y(t) = e^{-t} - e^{-2t}$$

where  $t$  is the time in hours since the injection.

Spock knows that if the concentration reaches 0.25 ppm or more, he will die.

Does Spock survive? Or does "Bones" McCoy (the ship's doctor) get to utter his famous line, "He's dead, Jim!" to Captain Kirk yet one more time? *Check the appropriate box on the answer sheet; justify your conclusion with some appropriate calculations. [Note: an answer—even the correct answer—without supporting calculations will receive little or no credit!]*

- b) Meanwhile, Captain Kirk has ordered coffee from the Yeoman of the watch, but it is too hot to drink. Suppose the coffee is at  $90^\circ\text{C}$  and arrives at exactly the instant that Spock is poisoned. Suppose that 6 minutes (i.e., 0.1 hr) later, the coffee has cooled to  $60^\circ\text{C}$ . Kirk can drink the coffee when it has cooled to  $30^\circ\text{C}$ . Does Kirk get to drink his coffee before saving Spock or must the coffee wait (i.e., does the coffee cool to  $30^\circ\text{C}$  before the poison reaches a lethal dose in Spock's blood (if it does))? *Write "yes" or "no" on your answer sheet, then justify your answer by writing and solving an appropriate equation.*

2. Consider the differential equation  $x \frac{dy}{dx} - 4y = x^6 e^x$  subject to the initial condition  $y(1) = 2$ . Solve this equation. *Be sure to write your answer to each part of this problem in the appropriate space on the answer sheet.*
- Write the equation in standard form (2 pts.)
  - Find the integrating factor (3 pts.)
  - Solve the equation subject to the initial condition (5 pts.)

**Part 3: Maple**

1. (2 pts) Which of the following Maple statements computes the value of  $\int_0^2 \frac{1}{4+x^2} dx$

a) integrate (1/(4+x^2), x = 0..2);

b) int (1/4+x^2, x = 0..2);

c) Int (1/(4+x^2), x = 0..2);

d) int (1/(4+x^2), x = 0..2);

e) integrate (1/4+x^2, x = 0..2);

f) diff (1/4+x^2, x = 0..2);

g) diff (1/(4+x^2), x = 0..2);

h) int (1/(4x^2), 0..2);

2. (3 pts) Write a Maple command to plot both  $y = -2\sinh(x)$  and  $y = \frac{1}{2}e^{-x^2}$ ,  $-3 \leq x \leq 5$ , on the same graph.  
[BE SURE TO WRITE YOUR COMMAND ON THE ANSWER SHEET! Use more than one line if necessary]