

1. Find an equation for the tangent line to the curve $x = \frac{1}{t}$, $y = -2 + \ln t$ at $t = 1$.

Answer: $y = -x - 1$

2. Find the slope of the curve given by $x \sin t + 2x = t$, $t \sin t - 2t = y$ at $t = \pi$

Answer: -4

3. Find the length of one arch of the cycloid

$$x = a(t - \sin t), y = a(1 - \cos t).$$

Answer: $8a$

4. Find the area of the surface generated by revolving one arch of the cycloid

$$x = t - \sin t, y = 1 - \cos t$$

about the x-axis.

Answer : $\frac{64\pi}{3}$

5. Replace the following polar equations by equivalent Cartesian equations:

a) $r = \cos^3 \theta - \sin \theta$

b) $r = 4 \tan \theta \sec \theta$

6. Replace the following Cartesian equations by equivalent polar equations:

a) $x^2 + xy + y^2 = 1$

b) $(x + 2)^2 + (y - 5)^2 = 16$

7. Find the area shared by $r = 1$ and $r^2 = 2 \sin 2\theta$.

8. Find the area of the surface generated by revolving the curve $r^2 = \sin 2\theta$ about the y-axis.

Answer: 4π

9. Find a vector 5 units long in the direction opposite to the direction of $\vec{a} = 5\vec{i} - 3\vec{j} - 4\vec{k}$.

Answer: $-\frac{5}{\sqrt{50}}\vec{i} + \frac{3}{\sqrt{50}}\vec{j} + \frac{4}{\sqrt{50}}\vec{k}$

10. Find the unit vectors which are tangent and normal to :

a) $3x^2 + 8xy + 2y^2 - 3 = 0$, at $(1,0)$

b) $y = \int_e^x \ln(\ln(t))dt$, at $(e, 0)$.

11. Describe(with a single equation or a pair of equations) the set of points in space equidistant from the points $(2, 0, 4)$ and $(1, 3, -1)$.

Answer: $-x + 3y - 5z = -\frac{9}{2}$

12. What is the center of the sphere given by the equation $3x^2 + 3y^2 + 3z^2 + 2y - 2z = 9$?

Answer: $(0, -\frac{1}{3}, \frac{1}{3})$

13. Find the equation of the plane that passes through $(3, -2, 1)$ and it is normal to the vector $\vec{n} = 2\vec{i} + \vec{j} + \vec{k}$.

Answer: $2x + y + z = 5$

14. If $|\vec{v}| = 2$, $|\vec{w}| = 3$ and the angle between \vec{v} and \vec{w} is $\pi/3$ find $|\vec{v} - 2\vec{w}|$.

Answer: $2\sqrt{7}$

15. For what values of a are the vectors $\vec{u} = 2\vec{i} + 4\vec{j} - 5\vec{k}$ and $\vec{v} = -4\vec{i} - 8\vec{j} + a\vec{k}$ parallel?

Answer: $a=10$

18. True or false:

a) $\vec{u} \cdot \vec{u} = |\vec{u}|$

b) $(\vec{u} \times \vec{v}) \cdot \vec{w} = (\vec{v} \times \vec{w}) \cdot \vec{u}$

c) $(\vec{u} \times \vec{v}) \cdot \vec{u} = (\vec{v} \times \vec{u}) \cdot \vec{v}$

Answer: a) false, b)true, c)true