1 Homework solutions from section 7.2

33) We are given the function $f(u) = e^{\frac{1}{u}}$ and we want to find the derivative of $f(u)$. Using the chain rule and setting $v = \frac{1}{u}$, we obtain that $\frac{df}{du} = \frac{df}{dv}\frac{dv}{du} = \frac{-1}{u^2}e^{\frac{1}{u}}$ where $f(v) = e^v$.

73) We want to integrate the expression $\int e^x\sqrt{1+e^x}dx$.
We proceed by making the substitution $u = 1 + e^x$. Now taking the derivative wrt to x, we obtain $\frac{du}{dx} = e^x$. We now make the substitution $du = e^x dx$ in the integrand. The integral now becomes $\int e^x\sqrt{1+e^x}dx = \int \sqrt{u}du = \frac{2}{3}u^{\frac{3}{2}} + c = \frac{2}{3}(1+e^x)^{\frac{3}{2}} + c$ where c is a constant of integration.

2 Additional problems

1) We want to find the volume of the solid of revolution generated by rotating about the y axis the region enclosed by the graph of $y = e^{-x^2}$, the x axis and the lines $x = 0$ and $x = 2$.
Using cylindrical shells, we have that the volume of the solid is given by $V = \int_0^2 2\pi x e^{-x^2} dx$. We solve this using the substitution $v = x^2$ and this gives the volume $V = \pi(1 - e^{-4})$.

2) We want to find the slope of the function $f(x) = e^{2xcosx}$ at the point $(0, 1)$. We differentiate $f(x)$ to obtain $f'(x) = e^{2xcosx}[2cosx - 2xsinx]$. At the point $(0, 1)$, the slope is $f'(0) = 2$. Hence the equation of the tangent line is $y = 2x + 1$.

3) We are given the function $f(x) = \frac{2-\sqrt{x}}{3+\sqrt{x}}$ and we want to determine its inverse function f^{-1}. Let $y = \frac{2-\sqrt{x}}{3+\sqrt{x}}$. Interchanging x and y we have $x = \frac{2-\sqrt{y}}{3+\sqrt{y}}$. Now solving for y iin terms of x, we obtain $y = \left(\frac{3x-2}{x+1}\right)^2$. Hence $f^{-1} = \left(\frac{3x-2}{x+1}\right)^2$.

4) We are given that the function $f(x) = x^6 + x^5 + 2$ is one to one. We want to determine $f^{-1}(4)$. By definition, $f^{-1}(4) = \frac{1}{f'(f^{-1}(4))}$. We further observe that $f'(x) = 6x^5 + 5x^4$. Now $f^{-1}(4)$ is the value of x for which $x^6 + x^5 + 2 = 4$ by the definition of the inverse function. By inspection, we obtain $x = 1$ is a solution. Hence $f^{-1}(4) = 1$. This implies that $f'(f^{-1}(4) = 6 + 5 = 11$. This then gives us $f^{-1}(4) = \frac{1}{f'(f^{-1}(4)} = \frac{1}{11}$.