Homework 5 *Math 361, Fall 2007*

1. Prove that if $f^{-1}((a,\infty))$ is measurable for every $a \in \mathbb{R}$, then f is a measurable function (according to Definition 33 in the Lecture Notes).

2. Prove Proposition 39 in the Lecture Notes.

   Let $f_1, f_2, \ldots$ be a sequence of measurable functions in $\mathbb{R}^n$. If the pointwise limit

   $$f(x) = \lim_{i\to\infty} f_i(x)$$

   exists for every $x \in \mathbb{R}^n$, then f is a measurable function.

3. Consider the *Fourier Series*

   $$A_0 + \sum_{n=1}^{\infty}(A_n \cos nx + B_n \sin nx),$$

   familiar from math 241. Suppose the coefficient A_n, B_n are such that the series converges *pointwise*. This means that for every specific value of x, the series converges to a finite real number which we will denote as $f(x)$. Then the series defines a function.

   Pointwise convergence is a much weaker condition than *uniform convergence*. If the series converges *uniformly*, then f is a continuous function (see Chapter 5 in the text book).

   Prove that if the series converges *pointwise*, then the function f is measurable. Use the propositions from section 2.2 in the Lecture Notes.