Remark. The area of region R in the xy-plane is obtained as the double integral

$$\text{Area } R = \iint_R 1 \, dA,$$

which means that you must integrate the constant function $f(x, y) = 1$ over the region R. Likewise, the volume of a spatial region E is obtained as the triple integral

$$\text{Volume } E = \iiint_E 1 \, dV.$$

Explanation: You know that the *average* of any function f over the region R equals $\iint_R f \, dV$ divided by the area of R. But for the specific function $f(x, y) = 1$ the average of f is of course 1 (because f is constant). Therefore the integral $\iint_R 1 \, dA$ divided by the area of R equals 1, which is another way of saying that the area of R equals $\iint_R 1 \, dA$.

1. Find the volume of the region E inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $3z^2 = x^2 + y^2$ (with $z \geq 0$).

   A) π B) $\frac{2}{3}\pi$ C) $\frac{3}{2}\pi$ D) $\frac{8}{3}\pi$ E) 2π F) 3π

2. Find the area of the region R contained inside the ellipse

$$5x^2 - 8xy + 5y^2 = 9.$$

To evaluate the integral, make use of the change of variables,

$$x = 2u + v,$$
$$y = u + 2v.$$

A) π B) $\frac{2}{3}\pi$ C) $\frac{3}{2}\pi$ D) $\frac{8}{3}\pi$ E) 2π F) 3π

3. Find the mass of a body contained in the first octant whose boundary surfaces are the cone $3z^2 = x^2 + y^2$ and the three planes $x = 0$, $y = 0$, and $z = 1$. The mass density at (x, y, z) is equal to xyz.

A) 0 B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) $\frac{3}{16}$ E) $\frac{2}{9}$ F) $\frac{7}{32}$