

Homework questions for chapter 10

Math 114, Spring 2008

1. Sketch a direction field for the differential equation

$$\frac{dy}{dt} = y^3 - 4y.$$

Find the equilibrium solutions, and sketch some of the solution curves.

Based on your graph, decide which of the following statements is correct. (Select one answer.)

- A) If the initial value satisfies  $0 < y(0) < 2$ , then  $\lim_{t \rightarrow \infty} y(t) = 2$ .
- B) If the initial value satisfies  $0 < y(0) < 2$ , then  $\lim_{t \rightarrow \infty} y(t) = \infty$ .
- C) If the initial value satisfies  $-2 < y(0) < 2$ , then  $\lim_{t \rightarrow \infty} y(t) = -2$ .
- D) If the initial value satisfies  $-2 < y(0) < 2$ , then  $\lim_{t \rightarrow \infty} y(t) = 0$ .
- E) If the initial value satisfies  $y(0) < -2$ , then  $\lim_{t \rightarrow \infty} y(t) = 2$ .
- F) If the initial value satisfies  $y(0) < -2$ , then  $\lim_{t \rightarrow \infty} y(t) = -2$ .

2. If  $y(t)$  represents a sum of money in an account at time  $t$  with constant interest rate  $r_0$ . If interest is compounded continuously, the differential equation that governs the growth of the account is

$$\frac{dy}{dt} = r_0 y.$$

The solution of this equation shows that  $y(t)$  grows exponentially as  $y(t) = Ce^{r_0 t}$ .

Now suppose that  $w(t)$  represents a sum of money in a second account with a variable interest rate  $r$ . Initially the rate  $r$  on this second account is 1% less than the rate  $r_0$  on the first account, but it increases steadily with .5% per time unit. In summary,

$$r = r_0 - 0.01 + t \times 0.005.$$

The differential equation for the growth of  $w(t)$  needs to be adjusted accordingly,

$$\frac{dw}{dt} = rw = (r_0 - 0.01 + t \times 0.005)w.$$

Solve this differential equation and then answer the following question. If the two accounts have the same initial amounts  $y(0) = w(0)$ , then  $y(t)$  will initially grow faster than  $w(t)$  because of the higher interest rate. But eventually  $w(t)$  will be greater than  $y(t)$ . At what precise point in time is  $y(t) = w(t)$  (not including the initial moment  $t = 0$ )?

- A)  $t = 1$       B)  $t = \ln 2$       C)  $t = 2$       D)  $t = 2 \ln 2$       E)  $t = 4$       F)  $t = 4 \ln 2$

3. Solve the initial value problem

$$y' = 2xy - 6e^{x^2}$$
$$y(0) = 6.$$

What is the value of  $y(1)$ ?

- A) 0      B) 1      C)  $e$       D)  $e^4$       E)  $4 + e$       F)  $\frac{3e+1}{e}$