Multiple choice questions for Chapter 10 *Math 104, Spring 09*

Show your work. No partial credit is given, but a correct guess without supporting work receives no credit.

1. The population growth $P(t)$ of a collection of bacteria is modeled by the differential equation

$$\frac{dP}{dt} = 3P(2-P)(5-P).$$

For which *initial values* $P(0)$ does the population size eventually tend towards the equilibrium value $P = 2$? In other words, find *all* (nonnegative) initial values for which

$$\lim_{t\to\infty} P(t) = 2.$$

Hint. Sketch a direction field.

A.) For all initial values $P_0 > 0$

B.) For all initial values $0 < P_0 \leq 2$

C.) For all initial values $0 < P_0 < 5$

D.) For all initial values $2 \leq P_0 < 5$

E.) For all initial values $P_0 \geq 2$

F.) Only if $P_0 = 2$

2. A tank contains 1000 cubic feet of water in which 15 lbs of salt has been dissolved. Water containing 0.02 lbs of salt per cubic feet enters the tank at a rate of 10 cubic feet per minute. The solution is kept thoroughly mixed. Simultaneously, 10 cubic feet per minute of the solution is drained from the tank, so that the total content is kept at 1000 cubic feet at all times. Find a formula for the amount of salt S that is in the tank after t minutes.

   A.) $S = Ae^{0.02t}$

   B.) $S = 15 + e^{-0.02t}$

   C.) $S = 20 - 5\,e^{-0.01t}$

   D.) $S = 10 + 10\,e^{0.02t}$

   E.) $S = 15 - e^{0.01t}$

   F.) $S = 20 + 5\,e^{-0.02t}$

3. The function $y = e^x$ is the solution of the initial value problem

$$\begin{aligned} y' &= y \\ y(0) &= 1. \end{aligned}$$

Apply Euler's method to this initial value problem. If you use step size $\Delta x = 0.2$ which approximation do you get for $e = y(1)$?

Remark. Use a calculator. Show a table summarizing the steps of your calculation.

A.) $e \approx 2.218$

B.) $e \approx 2.398$

C.) $e \approx 2.488$

D.) $e \approx 2.578$

E.) $e \approx 2.718$

F.) $e \approx 2.978$