

Math 241, Fall 2004
Homework Assignment #2

1. Suppose $f(x) = x^2 + 1$ for $0 \leq x \leq 1$. Draw the graph of
 - (a) f_o the odd 2-periodic extension of f .
 - (b) f_e the even 2-periodic extension of f .
 - (c) f_p 1-periodic extension of f .
2. With the notation for the previous problem. What does the Fourier series of f_o converge to at $x = 0$? What about the Fourier series of f_e and f_p ?
3. Find the sine half range expansion of $f(x)$ where f is given in problem 1.
4. Find the cosine half range expansion of $f(x)$ where f is given in problem 1.
5. Let $f(x) = \sin x$ on $0 < x < \pi$. Compute the half-range cosine and sine series of $f(x)$.
6. If $f(x)$ and $g(x)$ are both odd, then show $h(x) = f \circ g(x)$ is an odd function.
7. If $f(x)$ is periodic of period π then what period is $g(x) = f(5x)$?
8. If $f(x)$ is defined over the interval $[0, 4]$, then what is the period of the odd periodic extension of f ?
9. Suppose $g(t) = \sum_{n=1}^{\infty} b_n \sin nt$. Consider the differential equation

$$y'' + 4y = g(t).$$

- (a) Assume $b_2 = 0$. Find the Fourier series of a particular solution to the equation.
 - (b) Assume $g(t) = \sin 2t$. Find the particular solution of the equation. (You might want to review the method of undetermined coefficients.)
 - (c) Don't write this up in your homework, but what can you say about the solution in case (a) and (b) for large t .
10. Let f be the 2π periodic function given by

$$f(x) = \begin{cases} -1 & \text{for } -\pi \leq x \leq 0 \\ 1 & \text{for } 0 < x < \pi \end{cases}$$

Find a solution to the following differential equation

$$y'' - y' - 6y = f(x).$$

Hint: We computed the Fourier series of $f(x)$ in class.

11. Solve the following Sturm-Liouville problem. (That is find the eigenvalues and eigenfunctions.)

$$y'' - \lambda y = 0, \quad 0 \leq x \leq L$$
$$y(0) = 0, y'(L) = 0$$