Math 241, Fall 2004
Homework Assignment #7

1. Using separation of variables find a bounded function u solving $\nabla^2 u = 0$ in the region $r > 7$ subject to $u(7,\theta) = f(\theta)$. (Be careful, the region is $r > 7$ not $r < 7$ as we did in class.)

2. Find the solution to $\nabla^2 u = 0$ on the region $r < 3$ and $0 \leq \theta \leq \frac{\pi}{2}$ subject to the boundary conditions $u(r, 0) = 0, u(r, \frac{\pi}{2}) = 0$ and $u(3, \theta) = f(\theta)$.

3. In Euclidean 3–space solve $\nabla^2 u = 0$ subject to $u(3, \phi, \theta) = \cos^2 \theta$.

4. Use the Laplace transform to solve
$$u_{xx} = u_{tt}, \quad 0 < x < 1, t > 0$$
$$u(0,t) = 0, \quad u(1,t) = 0$$
$$u(x,0) = 0, \quad u_t(x,0) = 2\sin 2\pi x + 4\sin 3\pi x.$$

5. Let
$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0. \end{cases}$$
   (a) Show $\mathcal{F}\{H(x)e^{-ax}\} = \frac{1}{a-iw}$ where $a > 0$.
   (b) Show $\mathcal{F}\{H(-x)e^{ax}\} = \frac{1}{a+iw}$ where $a > 0$.
   (c) Show $\mathcal{F}\{e^{-a|x|}\} = \frac{2a}{w^2+a^2}$, where $a > 0$.
   HINT: Use your previous two answers.

6. Show $\mathcal{F}\{\frac{1}{a}e^{i\frac{bx}{a}} f(\frac{x}{a})\} = \hat{f}(aw + b)$.

7. Show $\mathcal{F}\{f(ax)\cos cx\} = \frac{1}{2a}\left(\hat{f}(\frac{w-c}{a}) + \hat{f}(\frac{w+c}{a})\right)$, where $a > 0$.
   HINT: Recall $cos\, cx = \frac{1}{2}(e^{icx} + e^{-icx})$. Now use the previous problem.

8. Show $\mathcal{F}\{f(x-a)\} = e^{iaw}\hat{f}(w)$.

9. Using the Fourier transform find $u(x,y)$ solving
$$\nabla^2 u = 0, \quad -\infty < x < \infty, 0 < y < \infty,$$
$$u(x,0) = f(x).$$
   You may assume that $u \to 0$ and $u_x \to 0$ as $x \to \pm\infty$ and $u \to 0$ as $y \to \infty$. (The first two limits are necessary to apply the Fourier transform, the last limit will help in the problem.)