Math 241, Fall 2004
Homework Assignment #8

1. Study for the second midterm.

2. Using the Fourier transform find $u(x, y)$ solving

$$\nabla^2 u = 0, \quad -\infty < x < \infty, 0 < y < \infty,$$

$$u(x, 0) = f(x).$$

   You may assume that $u \to 0$ and $u_x \to 0$ as $x \to \pm\infty$ and $u \to 0$ as $y \to \infty$. (The first two limits are necessary to apply the Fourier transform, the last limit will help in the problem.)

3. Find all the 3^{rd} roots of 1. Plot these roots in the complex plane.

4. Find all the 6^{th} roots of 1. Plot these roots in the complex plane.

5. Find all the 5^{th} roots of $1 + i$. Plot these roots in the complex plane.

6. Find $\sqrt{2 + 3i}$.

7. Solve $z^2 + z - \frac{i}{4} = 0$. (The quadratic formula is true for complex numbers too.)

8. Describe in words the following regions.

   (a) $|z - 2| = 4$.
   (b) $|z + (2 - i)| \leq 9$.
   (c) $|z + i| \geq 1$.
   (d) $0 < |z - 1| \leq 5$.

9. Prove the angle sum identities

$$\cos(\theta + \phi) = \cos\theta \cos\phi - \sin\theta \sin\phi$$

$$\sin(\theta + \phi) = \cos\theta \sin\phi + \sin\theta \cos\phi$$

   using the fact that $e^{ix} = \cos x + i \sin x$.