Math 241, Fall 2004
Homework Assignment #9

1. Let $f(z) = ze^z$

   (a) Find the real and imaginary parts of $f(z)$.

   (b) Verify the real and imaginary parts satisfy the Cauchy-Riemann equations.

2. If $f(z) = u(x,y) + i\,v(x,y)$ then $f_x = u_x(x,y) + i\,v_x(x,y)$ and $f_y = u_y(x,y) + i\,v_y(x,y)$. Show the Cauchy-Riemann equations are equivalent to

$$f_x + i\,f_y = 0.$$

3. Suppose f, g are analytic functions in the domain D. If $\mathrm{Re}(f) = \mathrm{Re}(g)$ in D show that $f(z) = g(z) + c$ where c is some constant.
   HINT: Consider $h = f - g$ and thing about the Cauchy-Riemann equations.

4. Suppose $f(z)$ and $\overline{f(z)}$ are analytic in the domain D. Show that f must be constant in D.
   HINT: Consider $h(z) = f(z) - \overline{f(z)}$ are try to use the last problem.

5. Show $x^2 - y^2 - xy$ is harmonic and find it's harmonic conjugate.

6. Show that if $u(x,y)$ and $u^2(x,y)$ are both harmonic functions then u is constant.
   HINT: Consider $\nabla^2\, u^2(x,y)$ and show that $u_x^2 + u_y^2 = 0$.