

Math 241 Practice Midterm

Name: _____

Student ID: _____

Signature: _____

Instructions: Print your name and student ID number and sign your signature to indicate that you accept the honor code. You may use **one page** of notes on this test. You may not use any other notes, books, calculators or computers. When a box is provided for your answer you must write your answer (and nothing else) in the box to receive full credit for the problem. Even if the correct answer appears somewhere else on the page, you will not receive full credit. Moreover, you must also **show the work** you did to arrive at the answer to receive full credit. You have 1.5 hours to answer all the questions. *Good Luck*

Question	Max Point	Score
1	6	
2	4	
3	5	
4	5	
5	3	
6	3	
7	5	
8	4	
9	5	
Total	40	

1) True or False: (1 point each) Circle **T** for True and **F** for False.

1. All functions have a convergent Fourier series.

T **F**

2. The functions $\sin nx$, n a positive integer, form an orthogonal set over $[-\pi, \pi]$ (with respect to the weight function 1).

T **F**

3. A function cannot be both even and odd.

T **F**

4. If $f(x)$ is an odd function and $g(x)$ is an even function then $f(g(x))$ is odd.

T **F**

5. If a function f has period L then it cannot have period P for any $P \neq L$.

T **F**

6. Two different functions can have the same Fourier series.

T **F**

2)Short Answer: (1 point each)

1. Suppose the Fourier Series for a periodic function f with period 2π has $a_n = 0$ for $n = 0, 1, 2, \dots$. What is $\int_{-\pi}^{\pi} f(x) dx$?

Answer:

2. Suppose $f(x)$ has period 4 and $f(x) = x$ for $-2 < x < 2$, and its Fourier series is $a_0 + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi}{2}x + b_n \sin \frac{n\pi}{2}x)$. Find a_n .

Answer:

3. If $f(x)$ and $g(x)$ have period 2π and f has Fourier coefficients a_n and b_n and g has Fourier coefficients a'_n and b'_n then what are the Fourier coefficients of $f + g$?

Answer:

4. Without actually solving the equation, what radius of convergence can you guarantee for a power series solution about $x = 0$ to the ordinary differential equation $(x^2 - 1)y'' + (x + 2)y' - (x - 8)y = 0$.

Answer:

3) (5 points) Find a solution $y(x)$ of the differential equation

$$y'' - y' - 6y = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} \sin nx.$$

Answer:

4) (5 points) Suppose $f(x)$ has period 4 and $f(x) = x$ for $-2 < x < 2$, and its Fourier series is $a_0 + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi}{2}x + b_n \sin \frac{n\pi}{2}x)$. Find a_n and b_n .

Answer:

5) (3 points) Suppose $f(0) = g(1) = 0$ and f^2 and g' are orthogonal on $[0, 1]$ (with respect to the weight function 1, *i.e.* $\int_0^1 f^2 g' dx = 0$). Are the function $f f'$ and g orthogonal? Justify your answer. **HINT:** Integrate by parts.

Answer:

6) (3 points) Let y_n , for n a positive integer, be an orthogonal set of functions on $[1, 2]$ with respect to $p(x) = x$. Assume $\int_1^2 x y_n^2(x) dx = 3$ for all n . If $\ln x = \sum_{n=1}^{\infty} \frac{1}{n^2} y_n(x)$ (and the convergence is uniform) then compute

$$\int_1^2 x y_8(x) \ln x dx.$$

Answer:

7) (5 points) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem $y'' + \lambda y = 0$ subject to the boundary conditions $y'(0) = 0, y'(3) = 0$.

Answer:

8) (4 points) Let $f(x) = x^2 - 1$ for $0 \leq x \leq 3$. Draw the odd periodic extension f_o of f . Draw the even periodic extension f_e of f . Draw the period 3-periodic extension f_p of f . What does the Fourier series of f_e converge to at $x = 0$? What does the Fourier series of f_o converge to at $x = 0$? What about at $x = -2$? (The graphs should not go in the answer box, but make sure they are labeled appropriately.)

Answer:

9) (5 points) Find two independent power series solutions to the equation

$$(x - 1)y'' + y' = 0.$$

You should be able to find nice formula's for your solutions.

Answer: