

Oral Exam Questions - Asher Auel (2006)

Algebraic Number Theory (Harbater)

Give an example of a cyclic extension of \mathbb{Q} of degree 5. (I gave the maximal real subfield of $\mathbb{Q}(\zeta_{11})$.)

Where is this extension ramified?

Are all extensions of \mathbb{Q} ramified? Are all extensions of number fields ramified?

Give an example of an unramified extension of number fields. (I gave a quadratic example, $\mathbb{Q}(\sqrt{-5})$ I think.)

What can you say in general about unramified extensions of number fields? Are there infinitely many? Are they all abelian?

In your quadratic example, how many primes split in the extension? (I first calculated exactly which primes split, and noted that by Dirichlet's theorem they split with density $1/2$, but then I told them that in general you get the same thing using Chebotarev.)

What can you say about the ramification of primes in an A_5 extension of \mathbb{Q} ? Specifically, what can you say about inert primes and totally ramified primes? (Hint: consider higher ramification groups.)

Algebraic Number Theory (Chinburg)

Back to your cyclic of degree 5 field. You said it's unramified outside 11. What's the maximal pro-5 extension of \mathbb{Q} unramified outside of 11? (Hint: consider the maximal abelian subextension of this thing and use the Burnside Basis Theorem.)

Algebraic Topology (Block)

What topological spaces have the closed disk as universal cover?

What surfaces can cover a surface of genus 2? genus 3? Is the Euler characteristic the only obstruction, i.e. do surfaces of all Euler characteristic permitted genera actually occur as covers?

Prove that the Euler characteristic is multiplied by the degree in a cover.

What can you say about simply connected closed 3-manifolds?

Compute the cohomology ring of the loop space of $\mathbb{C}P^3$. (Hint: use the Leray-Serre spectral sequence for the associated path-space fibration.)

Algebraic Number Theory (resumed, Chinburg)

Back to your cyclic of degree 5 field. What's the rank of the unit group? Find some interesting units.

Can you state the functional equation for the zeta function of this field? (I couldn't.)

Can you state the functional equation for the Riemann zeta function?

Write down the regulator of this field. In general, why is the regulator well-defined?

Factor the zeta function of this field into Artin L -functions. How can you reinterpret these Artin L -functions as Dirichlet L -series? Write out their Euler product and Dirichlet series expansion. (Hint: via the reciprocity map of CFT.)