Oral Exam Questions - Andrew Bressler (2006)

This is the bulk of the questions I was asked. There are probably a few follow-up questions that I am forgetting.

Enumerative Combinatorics

1. Determine the number of ways in which a regular n-gon can be divided into $n-2$ triangles, where different orientations are counted separately.

2. Determine the number of ways in which the interval $[0, n]$ can be covered by matchsticks of lengths 1 and 2. How would the generating function change if matchsticks of lengths 3, 6, and 12 were allowed as well? What if one were allowed different kinds of matchsticks of each length?

3. Explain the difference between compositions and partitions. How would one use restricted partitions to determine the number of ways one can make change for a certain amount of money? What are the asymptotics of the above generating function? How would one tailor the above generating function to count also by the number of coins used?

4. How many rooted binary trees with n internal nodes are there such that each node either 1) is a leaf or 2) has exactly 2 progeny?

Asymptotics of Multivariate Generating Functions

1. Determine the generating function $F(x, y, z) = \sum_{r,s,t} a_{r,s,t} x^r y^s z^t$ of the number of paths from the origin to the point (r, s, t) using steps $(0, 0, 1)$, $(0, 1, 0)$, $(0, 0, 1)$ and $(1, 1, 1)$. Step through the process of determining asymptotics for the above generating function, defining all terms as they are used.

2. Consider the generating function $e^z \sqrt{1-z}/(2-z) = F(z) = \sum_{n \geq 0} a_n z^n$. Give a preliminary estimate of the asymptotics using elementary analysis. Apply Darboux's Theorem to get a possibly better estimate of the main term asymptotics. State the Transfer Theorems of Flajolet and Odlyzko. Can they be applied to the above gen generating function? If so, do so.

Representation Theory of Compact Groups

1. Define the Fourier transform of a finite nonabelian group, defining terms as they are used. Does $\hat{G}$ (the set of isomorphism classes of irreducible representations) have a group structure? If so, what is its relationship to G? If Π_λ and $\Pi_{\lambda'}$ are different representations of the same isomorphism class, what is the relationship between $\Pi_\lambda(g)$ and $\Pi_{\lambda'}(g)$?

2. Let p be in $[0, 1]$ and $q = 1 - p$. Suppose starting at $t = 0$ with h in G, at each second $h(t)$ is multiplied on the right by g_1 in G with probability p and multiplied on the right by g_2 in G with probability q. Determine whether or not this system reaches a steady state, using the Fourier transform.