

Oral Exam Questions - Andrew Obus (2006)

Differential Geometry

Given a connected, smooth curve in \mathbb{R}^3 , parameterized by arc length such that at any point, either the torsion or the curvature is zero, must the curve lie in a plane? What if the curve is analytic (i.e., given by power series)?

Define cut locus. What are the cut loci of $L(5, 1)$ and $L(5, 2)$? Can two non-homeomorphic closed, connected surfaces have the same cut locus? Can any such surface have cut locus $S^1 \vee S^1 \vee S^1$?

Does $S^1 \times S^2$ have a Riemannian metric of constant positive sectional curvature? What about constant positive Ricci curvature? What about constant positive scalar curvature? What 4-manifolds have constant positive sectional curvature? Give an example of a 5-manifold with constant positive sectional curvature which is not homeomorphic to S^5 or $\mathbb{R}P^5$.

Algebraic Geometry

Consider the curve $y^2 = \prod_{j=1}^{10} (x - j)^j$. Tell me everything you can about this curve geometrically, over the reals and the complexes. Is it a complex manifold if we consider it over \mathbb{C} ? Can you remove several points to make it a complex manifold? Now, can you compactify it by adding in a finite number of points so that is still a complex manifold? What is the genus of this new manifold? What can you say about the rational points over \mathbb{Q} ? If q is large enough, must this curve have an \mathbb{F}_q -rational point?

Why is compactness not a good notion to use in the category of varieties (why is any affine variety compact)? What is the better notion? Why is this notion equivalent to the complex points forming a compact set (not necessarily a manifold) in the strong topology?

Can a curve of genus 7 be a branched cover of a curve of genus 6?

Can a curve of genus 11 cover a curve of genus 5? What if we allow branch points? How many non-isomorphic ways can we do this?

Are any 2 elliptic curves isogenous? Is isogeny an equivalence relation? How many isogeny classes of elliptic curves are there? What are some examples of finite morphisms $f : X \rightarrow Y$ which are not isomorphisms? Can you give an example where X and Y are not isomorphic?

Given a curve and a finite set of points, is there always a rational function with poles of specified orders at these points? In general, can you have a function with just a simple pole at *some* point? What about a double pole?