

Oral Exam Questions - Shea Vick (2006)

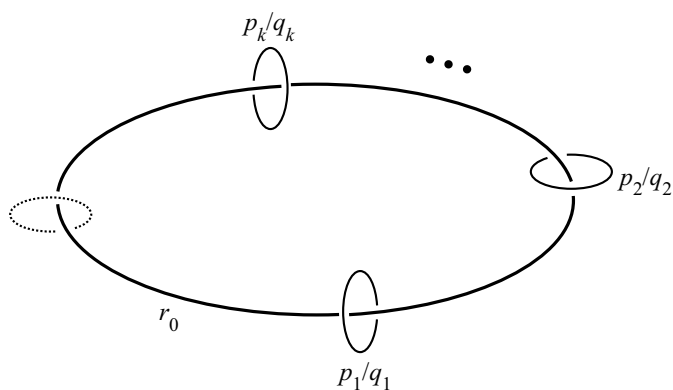
Algebraic/3-Manifold Topology

Describe a cell decomposition of $\mathbb{C}\mathbb{P}^2$. Compute the homology and cohomology [ring]. Does $\mathbb{C}\mathbb{P}^2$ have an orientation-reversing homeomorphism? Describe an $S^2 \subset \mathbb{C}\mathbb{P}^2$ with one algebraic self-intersection.

Define what it means for a 3-manifold M to be Haken. Describe an algebraic condition on M that guarantees that it's Haken... with proof!

Let $\Sigma = T^2 - (\text{open disk})$. Show that $\Sigma \times I$ is a handlebody. Do the same with $\Sigma_g - (k \text{ open disks})$. Do the same with $\mathbb{R}\mathbb{P}^2$, but thickened instead of $\times I$.

Describe a surgery you can do on the dotted circle so that the resulting manifold is a connected sum of lens spaces.



Show that, for a knot K in S^3 , $\pi_1(S^3 - N(K)) \cong \mathbb{Z}$ iff K is the unknot.

Differential Geometry

Does $SL(2, \mathbb{R})$ have a bi-invariant metric? Compute its Lie algebra.

What is a Jacobi field and what is it measuring?

Derive the Jacobi equation.

Describe the Jacobi fields on $S^2(1)$ (with proof).

Can a manifold with $K \leq 0$ have conjugate points? Prove that \exp is a local diffeomorphism.