DEPARTMENT OF MATHEMATICS

ORAL EXAMINATION

Major Area: PARTIAL DIFFERENTIAL EQUATIONS

1. Elementary methods and problems: Separation of variables for the wave, heat, and Laplace equations in standard regions. Typical initial and boundary value problems with elementary results for these problems.

2. Classical second-order operators of Mathematical Physics: Classification of linear operators, typical well-posed problems for the wave operator (hyperbolic case) -- finite propagation speed, energy estimates, Huygen's principle heat operator (parabolic case) -- fundamental solution, smoothness properties, maximum principle Laplace operator (elliptic case) -- basic properties of harmonic and subharmonic functions. Mean value properties, Liouiville theorem, smoothness, maximum principle, Harnack inequality, and Green's function for the sphere. Perron's method.

3. Generalized Functions: Distributions and Sobolev spaces (including embedding and compactness theorems), idea of weak solutions.

4. Second-Order Elliptic Equations: Typical problems (Dirichlet, Neumann, compact manifolds, etc.) maximum principle, Fredholm alternative, basic Schauder and L^p estimates.

5. This section has no title:
   (a) Analytic equations - Cauchy-Kovalevskaya theorem
   (b) Characteristics - classification of nonlinear equations
   (c) Weyl's asymptotic formula for the spectrum of the Laplacian

6. Nonlinear Functional Analysis: Compact mappings, Schauder fixed point theorem. Leray-Schauder degree theory with application to simple nonlinear elliptic PDE. Banach Space inverse and implicit function theorems. The statement and outline of the proof of the Nash-Moser inverse function theorem.

References:

D. Colton, Partial Differential Equations, an Introduction, Random House

G. Folland, An Introduction to Partial Differential Equations, Princeton U. Press.

D. Gilbarg & N.s Trudinger, Elliptic Partial Differential Equations of Second Order, Springer-Verlag.

L. Nirenberg, Topics in Nonlinear Functional Analysis, New York University Press.

R. Hamilton, Nash-Moser Inverse Function Theorem, Bull. of the AMS July 1982.