

## ORAL EXAMINATION

### **Minor area: Partial Differential Equations**

**General:** One should be prepared to exhibit serious understanding in depth with typical elementary examples illustrating the ideas. It is our experience that students sometimes know a smattering of advanced topics superficially but are unprepared to discuss the most basic elementary aspects of PDE which are customarily covered in undergraduate courses. Learn the elementary material FIRST.

**1.Elementary methods and problems:** separation of variables and Fourier/Laplace transform techniques for the wave, heat, and Laplace equations in standard regions; typical initial and boundary value problems with elementary results for these problems. One should also know the basic models in physics for the wave, heat, and Laplace equations.

**2.Classical Second-order Linear Operators of Mathematical Physics:** classification, typical well-posed problems, wave operator (hyperbolic case), finite propagation speed, energy estimates, Huygens' principle, wave propagation, heat operator (parabolic case), fundamental solution, smoothness properties, maximum principle, Laplace operator (elliptic case), properties of harmonic functions (such as the mean value property, Liouville theorem, smoothness, maximum principle, Harnack inequality, Greens' functions in simple regions), eigenfunctions and eigenvalues, variational principles.

**3.Generalized Functions:** Distributions, Sobolev spaces (including embedding and compactness theorems).

**4.Second Order Linear Elliptic Equations:** typical problems (Dirichlet, Neumann, on compact manifolds), maximum principle, Fredholm alternative.

### **Some References**

F. John, Partial Differential Equations, SpringerVerlag

G. Folland, Introduction to Partial Diff. Eq., Princeton U. Press