

DEPARTMENT OF MATHEMATICS

ORAL EXAMINATION

Minor area: ALGEBRAIC NUMBER THEORY

The material covered is from chapters 1 - 5 of "Number Fields" by D. Marcus:

- Chapter 1: Definitions of class numbers, ideal class groups, unique factorization of ideals. Connections between Fermat's last theorem and the Field $\mathbf{Q}(z_p)$.
- Chapter 2: Definitions of number fields, cyclotomic fields, quadratic fields, algebraic integers. Complex embeddings, real embeddings, trace, norm and discriminant. Simple examples.
- Chapter 3: Definitions of Dedekind rings, splitting of primes, ramification, and the different.
- Chapter 4: Decomposition and inertia groups, splitting of primes in subfields, primes that split completely, the Frobenius automorphism of a prime.
- Chapter 5: Finiteness of class numbers, structure of unit groups, volume of fundamental domain for the integers and for the units mod torsion. Minkowski's lemma and bounds for class numbers.