① $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n\,2^{n+3}} \Rightarrow \lim_{n\to\infty} \left| \frac{\frac{(2x+1)^{n+1}}{(n+1)\,2^{n+4}}}{\frac{(2x+1)^n}{n\,2^{n+3}}} \right| = \lim_{n\to\infty} \left| \left(x+\frac{1}{2}\right) \frac{n}{n+1} \right| = \left| x+\frac{1}{2} \right|$

So, $|x+1/2| < 1 \Rightarrow$ $x+\frac{1}{2} < 1 \Rightarrow x < \frac{1}{2}$

$x+\frac{1}{2} > -1 \Rightarrow x > -3/2$

At $x = \frac{1}{2}$; $\sum_{n=1}^{\infty} \frac{2^n}{n\,2^{n+3}} = \sum_{n=1}^{\infty} \frac{1}{8n}$, WHICH DIVERGES

At $x = \frac{-3}{2}$; $\sum_{n=1}^{\infty} \frac{(-2)^n}{n\,2^{n+3}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{8n}$, CONVEGERGES BY THE ALT^g SERIES TEST

$\Rightarrow I = \left[\frac{-3}{2}, \frac{1}{2}\right)$, F

② $f(x) = \frac{x}{x^2+2} + \frac{1}{1+x}$

$= \frac{x}{2}\left(\frac{1}{1-(-x^2/2)}\right) + \frac{1}{1-(-x)}$

$= \frac{x}{2}\sum_{n=0}^{\infty}\left(-\frac{x^2}{2}\right)^n + \sum_{n=0}^{\infty}(-x)^n$

$= \frac{x}{2}\left(1 - \frac{x^2}{2} + \frac{x^4}{4} + \cdots\right) + 1 - x + x^2 + \cdots$

$= 1 + \left(\frac{x}{2} - x\right) + x^2 - \frac{x^3}{4} - x^3 + \cdots$

$= 1 - \frac{x}{2} + x^2 - \frac{5}{4}x^3 + \cdots \Rightarrow$ F

③ $F(x) = \int_0^x \underbrace{\ln(t^2+1)}_{f(t)}\,dt$

FIND A SERIES EXPRESSION BY DIFFERENTIATING THE $\ln$ AND INTEGRATING TWICE

$\frac{\partial f(t)}{\partial t} = \frac{2t}{1+t^2} = 2t\sum_{n=0}^{\infty}(-t^2)^n$

$\int \frac{\partial f}{\partial t}\,dt = 2\sum_{n=0}^{\infty}(-1)^n \int t^{2n+1}\,dt$

$= 2\sum_{n=0}^{\infty}(-1)^n\left(\frac{t^{2n+2}}{2n+2} + c\right)$

TO FIX C, NOTE THAT AT $t=0$ $\ln(t^2+1) = 0$, SO $C=0$

$F(x) = \int_0^x 2\sum_{n=0}^{\infty}(-1)^n \frac{t^{2n+2}}{2n+2} = 2\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+2}\int_0^x t^{2n+2}$

$= 2\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+3}}{(2n+2)(2n+3)} = \frac{2x^3}{4} - \frac{2x^5}{20} + \frac{2x^7}{42} + \cdots \Rightarrow$ B