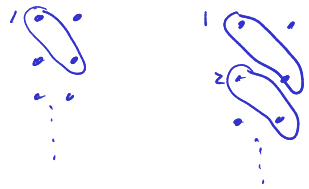
$|A| = 2\ell$, How many ways to pair points? $2\ell - 1$

Method 1: Pick a pt. It can be paired w/ another point in $2\ell-1$ ways.

Pick another pt. It can be paired in $2\ell-3$ ways.



Continue $\Rightarrow (2\ell-1)(2\ell-3)\cdots\cdot 3\cdot 1$

Method 2

Choose point 1 $\Rightarrow 2\ell$ } pair

Choose point 2 $\Rightarrow 2\ell-1$ } pair

Choose point 3 $\Rightarrow 2\ell-2$ } pair

Choose point 4 $\Rightarrow 2\ell-3$ } pair

$\Rightarrow (2\ell)!$

But, we over counted!

pairs can be exchanged in $\ell!$ ways, & in each pairing "the first" can be exchanged with "the second"

$$\Rightarrow \frac{(2\ell)!}{\ell!\, 2^\ell}$$

They are equal results

$$(2\ell-1)(2\ell-3)\times\cdots\times 1 = \frac{(2\ell)!}{2\ell(2\ell-2)\times\cdots\times 2}$$

$$= \frac{(2\ell)!}{2^\ell\, \ell(\ell-1)(\ell-2)\times\cdots\times 1}$$

$$= \frac{(2\ell)!}{2^\ell\, \ell!}$$