

3.2.3b

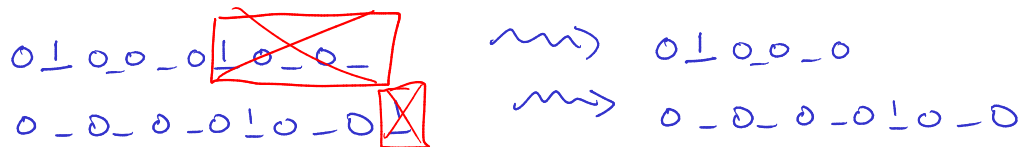


THE CONFIGURATION DESCRIBED IN THE PROBLEM

TO SEE IT GIVES j -TUPLES WHOSE SUM IS AT MOST n ,

1) INSERT j DIVIDERS IN j SLOTS. \Rightarrow $0 \perp 0 _ 0 \perp 0 _ 0 _$,
 $0 _ 0 _ 0 _ 0 \perp 0 _ 0 \perp$

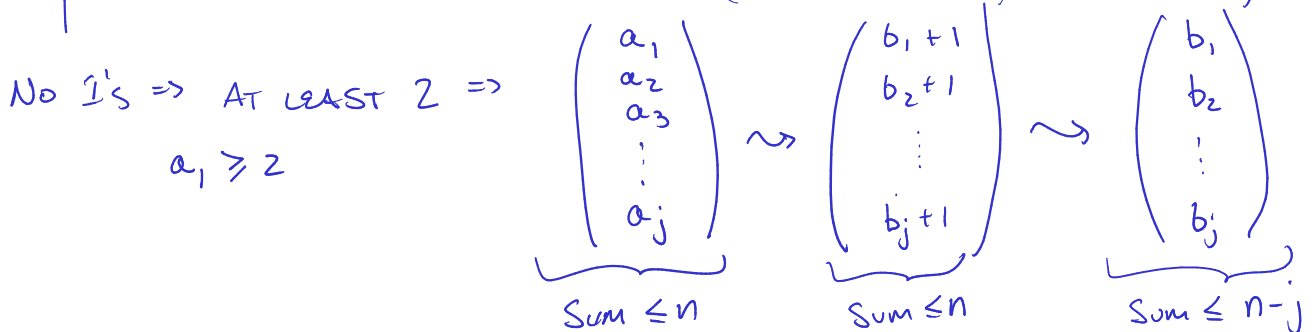
2) REMOVE THE LAST DIVIDER & BALLS TO THE RIGHT



3) WHAT REMAINS IS j GROUPS OF BALLS, WHOSE TOTAL # IS AT MOST n .

Now, how many ways? From n SLOTS, CHOOSE $j \Rightarrow \binom{n}{j}$

(b) REPHRASE AS (AT LEAST ONE 1) = (ALL ARRANGEMENTS) - (NO 1'S)



THUS, THE PROBLEM FOR THE a 'S CAN BE EXPRESSED AS A j -TUPLE OF POSITIVE INTEGERS WHOSE SUM IS AT MOST $(n-j)$. ACCORDING TO (a), THIS IS $\binom{n-j}{j}$.

$$\text{PROB} = \frac{\# \text{ of ways}}{\text{TOTAL}} = \frac{\binom{n}{j} - \binom{n-j}{j}}{\binom{n}{j}}$$