3.2.36

(a) $\underbrace{O\_O\_O\_\cdots O\_O\_}$

THE CONFIGURATION DESCRIBED IN THE PROBLEM

TO SEE IT GIVES j-TUPLES WHOSE SUM IS AT MOST n,

1) INSERT j DIVIDERS IN j SLOTS. $\Rightarrow$ $O\,|\,O\_O\_O\,|\,O\_O\_$, $O\_O\_O\_O\,|\,O\_O\,|$

2) REMOVE THE LAST DIVIDER & BALLS TO THE RIGHT

$O\,|\,O\_O\_O\,|\,O\_O\_$ $\rightsquigarrow$ $O\,|\,O\_O\_O$

$O\_O\_O\_O\,|\,O\_O\,|$ $\rightsquigarrow$ $O\_O\_O\_O\,|\,O\_O$

3) WHAT REMAINS IS j GROUPS OF BALLS, WHOSE TOTAL # IS AT MOST n.

NOW, HOW MANY WAYS? FROM n SLOTS, CHOOSE j $\Rightarrow \binom{n}{j}$

(b) REPHRASE AS (AT LEAST ONE 1) = (ALL ARRANGE^{t}S) − (NO 1's)

NO 1's $\Rightarrow$ AT LEAST 2 $\Rightarrow$ $a_1 \geq 2$

$$\underbrace{\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_j \end{pmatrix}}_{\text{Sum} \leq n} \rightsquigarrow \underbrace{\begin{pmatrix} b_1+1 \\ b_2+1 \\ \vdots \\ b_j+1 \end{pmatrix}}_{\text{Sum} \leq n} \rightsquigarrow \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_j \end{pmatrix}}_{\text{Sum} \leq n-j}$$

THUS, THE PROBLEM FOR THE a's CAN BE EXPRESSED AS A j-TUPLE OF POSITIVE INTEGERS WHOSE SUM IS AT MOST $(n-j)$. ACCORDING TO (a), THIS IS $\binom{n-j}{j}$.

$$\text{PROB} = \frac{\#\text{ of WAYS}}{\text{TOTAL}} = \frac{\binom{n}{j} - \binom{n-j}{j}}{\binom{n}{j}}$$