Math 116 Problem Set #2 Due Fri., Sept. 19, 2008, in class.

Read Apostol, Chapter 9, sections 1-7, pages 358-368; and Chapter 3, sections 1-10, pages 126-145. (In section 3.4, omit from Theorem 3.4 to the end of the section, pp. 134-135.) Optional: Also read sections 8-9 of Chapter 9, pages 368-371.

1. From Apostol, 9.6, page 365, do problems 1 (a,e,g), 3 (f,i), 6-8.

2. From Apostol, 3.6, pages 138-140, do problems 2, 27, 31.

3. a) Prove that for every complex number c and every positive integer n, there is a complex number z such that $z^n = c$. [Hint: If z has polar form (r, θ), what is the polar form of z^n?]

b) Explicitly find all complex numbers z such that $z^4 = -1$.

4. Using just the definition of limit, prove that $\lim_{x \to 0}(3x + 2) = 2$. In other words, for each $\varepsilon > 0$ show that there is an appropriate $\delta > 0$.

5. For each of the following functions f, determine whether $\lim_{x \to 0} f(x)$ exists; and if it does, find it. Explain carefully why your assertions hold.

a) $f(x) = 1$ for $x > 0$; $f(x) = -1$ for $x < 0$; and $f(0) = 0$. [This is called the *signum function*, often denoted $\mathrm{sgn}(x)$.]

b) $f(x) = |x|$.

c) $f(x) = x \sin(1/x)$ for $x \neq 0$, $f(0) = 0$.

d) $f(x) = x^2 \sin(1/x)$ for $x \neq 0$, $f(0) = 0$.

Note: You may find it helpful to graph these functions first. For parts (c) and (d), compare problem 27 in section 3.6; and consider the squeezing principle.