Math 116 Problem Set #5 Due Fri., Oct. 10, 2008, in class.

Reminder: The first exam will be held in class on Monday, October 6.

From Apostol, read Chapter 12, sections 7-16.

1. From Apostol, 12.8, pages 456-457: do problems 5, 6, 13, 19 (and explain the geometric interpretation in #19).

2. From Apostol, 12.11, pages 460-462: do problems 1, 10(a), 12, 17(a,b), 18.

3. From Apostol, 12.15, pages 467-468: do problems 1, 6, 7, 20.

4. In the situation of problem 6 on Problem Set #4, let $f(x) = \sin(\pi x)$ and $g(x) = \cos(\pi x)$. Evaluate $\|f\|$, $\|g\|$, and $f \cdot g$.

5. a) Show that if we define a new dot product on $\mathbb{R}^2$ by

$$(a_1, a_2) \cdot (b_1, b_2) = 2a_1b_1 + a_1b_2 + a_2b_1 + a_2b_2,$$

then the usual laws of dot product (see Apostol, Theorem 12.2) are still satisfied.

b) In terms of coordinates, write an explicit formula for a new norm on $\mathbb{R}^2$ that is related to this dot product by the equation $\|v\|^2 = v \cdot v$.

c) Explain why the properties of norm given in Theorems 12.4 and 12.5 are automatically satisfied for this new norm (i.e. without the need to do any new computations).

6. In $\mathbb{R}^n$ (with $n \geq 2$), show that there is *no* way to redefine the dot product in a way that will still satisfy the usual laws of dot product but will also give the norm defined in problem 18 in Apostol, section 12.11. [Hint: Under this norm, what are the norms of e_1, e_2, and $e_1 + e_2$, where $e_1, \ldots, e_n$ are the unit coordinate vectors?]