Math 116 | Problem Set #7 | Due Fri., Oct. 24, 2008, in class.

From Apostol, read Chapter 13, sections 9-23.

1. From Apostol, 13.11, page 487-488: do problems 1(c,d), 2(a), 3(c), 8(a).

2. From Apostol, 13.14, pages 491-492: do problems 1(a), 2, 3.

3. From Apostol, 13.21, page 503: do problems 4, 5, 8.

4. From Apostol, 13.24, pages 508-509: do problems 1, 7, 13, 26.

5. Suppose that $v, w \in \mathbb{R}^3$. If $v \times (v \times w) = 0$, what can you conclude about v and w? Is this a necessary and sufficient condition?

6. In analogy with Theorem 9.2 of Apostol, write a vector $(a, b, c, d) \in \mathbb{R}^4$ as $a+bi+cj+dk$. Also, define a multiplication law on $\mathbb{R}^4$ so that $ij = k$, $jk = i$, $ki = j$, $ji = -k$, $kj = -i$, $ik = -j$ (as with cross product); and also so that $i^2 = j^2 = k^2 = -1$ (unlike the cross product); and so that $1v = v = v1$ for all $v \in \mathbb{R}^4$.

a) Under this multiplication, evaluate $(i + j)^2$ and $(1 + i + j + k)(1 - i - j - k)$.

b) Which of the axioms of a field (Axioms 1-6 on page 18 of Apostol) are satisfied by the elements of $\mathbb{R}^4$ under vector addition and the above multiplication law?

c) Call an element $v \in \mathbb{R}^4$ *central* if $vw = wv$ for all $w \in \mathbb{R}^4$. Find all the central elements of $\mathbb{R}^4$.