Math 116 Problem Set #8 Due Fri., Oct. 31, 2008, in class.

From Apostol, read Chapter 14, sections 1-6.

1. From Apostol, 13.25, pages 509-511: do problems 1, 4, 13(a), 16.

2. From Apostol, 14.4, pages 516-517: do problems 2, 4, 8, 14, 15, 19.

3. From Apostol, 14.7, pages 524-525: do problems 1, 2, 7, 10, 17.

4. a) Let $L$ be a line in the plane and let $C$ be a conic section in the plane. At how many points can $L$ and $C$ meet? Give examples illustrating each possible value.

b) In part (a), if $L$ is tangent to $C$ at a point $P$, then at how many points (including $P$) can $L$ and $C$ meet?

c) Make a conjecture concerning the number of points at which two distinct conic sections $C, C'$ in the plane can meet. Give examples to illustrate each of the possible values.

5. Suppose that $F : \mathbb{R} \to \mathbb{R}^2$ is a differentiable vector-valued function, that $c \in \mathbb{R}$, and that $\int_c^x F(t)\,dt = (x^2 - x, x^2 - 1)$ for all $x \in \mathbb{R}$. Find $F$ and find $c$.

6. Let $F : \mathbb{R} \to \mathbb{R}^n$ be a differentiable vector-valued function that parametrizes the motion of a particle in $\mathbb{R}^n$ whose speed is always at most $c$ (where $c$ is some positive real number).

a) Prove that if $a < b$ then $\|F(b) - F(a)\| \leq c(b - a)$. Also explain why this is reasonable from a geometric point of view.

b) Give an example of a function $F$ and values $a < b$ for which there is equality in part (a), and give another example in which there is a strict inequality.