Math 116 Problem Set #10 Due Fri., Nov. 14, 2008, in class.

Reminder: The second exam will be held in class on Monday, November 10.

From Apostol, read Chapter 4, section 22.

1. From Apostol, 4.23, page 201, do problems 1, 2, 4, 10. In #1,2,4, also find all critical points (i.e. points where $f_x = f_y = 0$ or where either f_x or f_y is undefined). In #10, such a function is called *harmonic*.

2. Let $F : \mathbb{R} \to \mathbb{R}^3$ be a differentiable function such that F' is never 0. Let $T(t)$ be the unit tangent vector to the parametrized curve at the point $F(t)$. Show that the curvature $\kappa(t)$ is given by the formula $\|F''(t) \times F'(t)\|/\|F'(t)\|^3$ even if $T'(t) = 0$.

3. Let $a > b > 0$ and consider the ellipse $x^2/a^2 + y^2/b^2 = 1$. Let P, P' be the foci. Let $F : [0, 1] \to \mathbb{R}^2$ be a parametrization of the ellipse such that $F(0)$ lies on the x-axis and the speed is constant.

a) Show that $\|F(t) - P\| + \|F(t + \frac{1}{2}) - P\| = 2a$ for all $t \in [0, 1/2]$. [Hint: If Q lies on the ellipse, find the value of $\|Q - P\| + \|Q - P'\|$.]

b) Deduce that $\int_0^1 \|F(t) - P\| \; dt = a$.

c) Explain why this says that the average distance from a point on the ellipse to P is equal to a.

4. Let $F : \mathbb{R} \to \mathbb{R}^2$ be a differentiable function whose position and velocity vectors are linearly dependent at each point. Show that the curve traced out by F lies on a line through the origin. [Hint: If $F(t) = (f_1, f_2)$, then what is $(f_1/f_2)'(t)$?]