Math 116 Problem Set #11 Due Fri., Nov. 21, 2008, in class.

From Apostol, read Chapter 8, sections 1-6.

Note: Here are some references for derivatives and integrals of functions $f : \mathbb{R}^n \to \mathbb{R}$: Stewart, Calculus, 6th edition, Chapters 15 and 16; Apostol, Calculus, Volume II, 2nd edition, Chapters 8, 9, 11; Shurman, Multivariable Calculus, Chapters 4 and 6, available at http://people.reed.edu/~jerry/211/vcalc.html; Jones, Honors Calculus III/IV, Chapters 2, 3, and 9, available at http://www.owlnet.rice.edu/~fjones/.

1. From Apostol, 8.5, page 311, do problems 1-6.

2. From Apostol, 8.7, pages 319-320, do problems 1, 2, 4, 8, 9.

3. Let $a(x), b(x), c(x)$ be twice-differentiable functions $\mathbb{R} \to \mathbb{R}$.
a) Show that the set of solutions to the differential equation $y'' + a(x)y' + b(x)y = c(x)$ satisfies the axioms of a vector space (see §15.2) if and only if $c(x)$ is the constant function 0.
b) Show that if $a(x) = 0$, $b(x) = 1$, and $c(x) = 0$, then $\sin(x)$ and $\cos(x)$ are linearly independent elements of this vector space.

4. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = xy^2 - x^2 - y^2$.
a) Find all the critical points of f. For each of these points, determine whether it is a local maximum, a local minimum, or neither.
b) Determine whether the function f achieves a (global) maximum or minimum on $\mathbb{R}^2$.

5. Let $f(x, y) = x^2 - 4xy + y$. Find where f takes on its maximum and minimum values on the closed rectangle given by $0 \le x \le 2$, $0 \le y \le 1$. (You may assume that a continuous function achieves a maximum and a mimimum on any closed rectangle.) [Hint: First consider the interior of the rectangle. Then consider the four boundary edges.]

6. Let $f(x, y) = x^2 y$.
a) Find the volume under the graph of $z = f(x, y)$ over the rectangle in the (x, y)-plane with vertices $(0, 0)$, $(2, 0)$, $(0, 3)$, $(2, 3)$.
b) Do the same for the triangle with vertices $(0, 0)$, $(2, 0)$, $(2, 3)$.