From Apostol, read Chapter 8, sections 20, 21, 25, 27; and Chapter 15 (all).

1. From Apostol, 8.22, page 344, do problem 3; 8.24, page 347, do problem 3; 8.26, page 350, do problem 4; 8.28, page 355, do problem 2.

2. From Apostol, 15.5, page 555, do problems 1, 5, 13, 14; 15.9, page 560, do problems 7, 10, 14; 15.12, pages 566-568, do problems 3, 16(c); 15.16, page 576, do problem 1(b).

3. Fix a real number $c > 0$.

a) For which $v \in \mathbb{R}$ does the equation $\left(\frac{x}{c\cos v}\right)^2 - \left(\frac{y}{c\sin v}\right)^2 = 1$ define a hyperbola H_v with foci $(\pm c, 0)$? Does *every* hyperbola with these foci have this form?

b) Recall that the hyperbolic functions $\cosh t = (e^t + e^{-t})/2$ and $\sinh t = (e^t - e^{-t})/2$ satisfy $\cosh^2 t - \sinh^2 t = 1$ and $\cosh' = \sinh$, $\sinh' = \cosh$. For which $u \in \mathbb{R}$ does the equation $\left(\frac{x}{c\cosh u}\right)^2 + \left(\frac{y}{c\sinh u}\right)^2 = 1$ define an ellipse E_u with foci $(\pm c, 0)$? Does *every* ellipse with these foci have this form?

c) Show that $E_u \cap H_v = \{(\pm c\cosh u\cos v, \pm c\sinh u\sin v)\}$. [Hint: First show $\supset$. At how many points do E_u and H_v meet?]

4. In this problem we retain the notation of problem 3 above.

a) If we fix $v \in \mathbb{R}$, and let $(x, y) = (c\cosh u\cos v, c\sinh u\sin v)$, then what curve is parametrized as u varies? Find the corresponding velocity as a function of u.

b) If we fix $u \in \mathbb{R}$, and let $(x, y) = (c\cosh u\cos v, c\sinh u\sin v)$, then what curve is parametrized as v varies? Find the corresponding velocity as a function of v.

c) Find the angle at which the curves in (a) and (b) meet. [Hint: Take the dot product of the tangent vectors (or the velocity vectors) of the two curves.]

d) Show that the ellipses and the hyperbolas with foci $(\pm c, 0)$ form orthogonal trajectories. [Hint: You can use the previous parts instead of differential equations.]