Math 116 Problem Set #2 Due Fri., Sept. 25, 2009, in class.

Read Apostol, Chapter 1, sections 9-25, pages 61-83; and Chapter 3, sections 1-2, pages 126-130.

1. From Apostol, Chapter 1, section 1.15, page 70, do problems 1(b,c,f), 4, 5(a).

2. From Apostol, Chapter 1, section 1.26, pages 83-84, do problems 22(a), 23, 25. In problem 25, also verify this explicitly for the function x^6 and x^7.

3. From Apostol, Chapter 3, section 3.6, page 138, do problems 1, 3, 5, 11.

4. Define a function f by setting $f(x) = x$ if x is rational, and $f(x) = 0$ if x is irrational.

a) Determine whether f is integrable on the closed interval $[0, 1]$. If it is, evaluate the integral.

b) Determine whether $\lim_{x \to 1} f(x)$ exists. If it does, evaluate it.

c) Re-do part (b) for the limit as x goes to 0.

5. Using just the *definition* of limit, prove that $\lim_{x \to 0} (4x + 2) = 2$.

6. Define a function f on the closed interval $[0, 1]$ as follows: We take $f(0) = 0$. If $x = m/n$ is a positive rational number in lowest terms, we take $f(x) = 1/n$. If x is irrational then take $f(x) = 0$. Determine whether f is integrable on $[0, 1]$. If it is, evaluate the integral.

[Hint: Pick a positive integer n and then define a step function $t(x)$ as follows: $t(x) = 1/2$ on $[1/2 - 1/2^n, 1/2 + 1/2^n]$; $t(x) = 1/3$ on $[1/3 - 1/(2 \cdot 2^{n+1}), 1/3 + 1/(2 \cdot 2^{n+1})]$ and on $[2/3 - 1/(2 \cdot 2^{n+1}), 2/3 + 1/(2 \cdot 2^{n+1})]$; $t(x) = 1/4$ on $[1/4 - 1/(3 \cdot 2^{n+2}), 1/4 + 1/(3 \cdot 2^{n+2})]$ and on $[3/4 - 1/(3 \cdot 2^{n+2}), 3/4 + 1/(3 \cdot 2^{n+2})]$; etc. up through $t(x) = 1/n$ on appropriate intervals around the points having denominator n. For all other x define $t(x) = 1/n$. Show that t is an upper step function and evaluate its integral. Then consider what happens if n varies.]