Math 116 Problem Set #3 Due Fri., Oct. 2, 2009, in class.

Read Apostol, Chapter 3, sections 3-10, pages 130-145, and section 16, pages 150-152. Optional: Also read sections 12-14, pages 146-149, and sections 17-19, pages 152-154.

1. From Apostol, Chapter 3, section 3.6, page 139, do problems 27, 28, 32.

2. From Apostol, Chapter 3, section 3.11, page 145, do problems 2(a), 5.

3. From Apostol, Chapter 3, section 3.20, page 155, do problems 7, 8.

4. For each of the following functions f on $[0, 1]$, determine whether there is an $x \in [0, 1]$ such that $f(x) = 0$, and whether f attains a maximum on $[0, 1]$. In each case, relate your assertion to whether the Intermediate Value Theorem and the Extreme Value Theorem apply.
a) $f(x) = e^x - 2$
b) $f(x) = x^4 + x^2 + 1$
c) $f(x) = x + 1 - 3[x]$
d) $f(x) = 1/(2x^2 - 1)$ if x is rational, and $f(x)$ is undefined otherwise.

5. Which of the following functions on $[0, 3]$ are of the form $F(x) = \int_a^x f$ for some f and some a? For each that is not, explain why not. For each that is, give such an f and a.
a) F is defined by $F(x) = 0$ on $[0, 1)$, $F(x) = x - 1$ on $[1, 2)$, and $F(x) = 3 - x$ on $[2, 3]$.
b) F is defined by $F(x) = 1$ on $[0, 1)$, $F(x) = x$ on $[1, 2)$, and $F(x) = x - 1$ on $[2, 3]$.

6. Let f be the function given in problem 6 of Problem Set 2. For which real numbers a in $[0, 1]$ is the function f continuous at $x = a$? Prove your assertion.