Math 116 Problem Set #5 Due Fri., Oct. 16, 2009, in class.

Read Apostol, Chapter 4, section 22; and Chapter 12, sections 1-10.

1. From Apostol, 4.23, page 201, do problems 1, 2, 10. In #1 and 2, also find all critical points (i.e. points where $f_x = f_y = 0$ or where either f_x or f_y is undefined). In #10, such a function is called *harmonic*.

2. From Apostol, 12.4, pages 450-451, do problems 1(a,c,e), 2, 5, 12.

3. From Apostol, 12.8, pages 456-457: do problems 1(b,e), 4, 5, 13(a,b), 19.

4. From Apostol, 12.11, pages 460-462: do problems 1, 5, 12, 20. (In #20, we say that n-space together with this distance function is a *metric space*.)

5. For any complex number $z = x+iy$ with $x, y \in \mathbb{R}$, the *real part* of z is x and it is denoted by $\mathrm{Re}(z)$. For any positive integer n, define $f_n(x, y) = \mathrm{Re}((x + iy)^n)$, for all real numbers x, y. Are the functions f_1, f_2, f_3 harmonic? (See problem 1 above.) Any conjectures?

6. Let V be the set of continuous functions on the closed interval $[0, 1]$. We may add two functions by defining $(f + g)(x)$ to be $f(x) + g(x)$, and we may multiply a function by a real number by defining $(cf)(x) = c(f(x))$.

a) Show that the set V, under these two operations, satisfies all the vector laws (given in and just after Theorem 12.1, page 447 of Apostol, and also listed as axioms 1-10 in Section 15.2, pages 551-552).

b) Show that if we define $f \cdot g = \int_0^1 f(x)g(x)dx \in \mathbb{R}$, then the laws of dot product (listed in Theorem 12.2 of Apostol, page 451) are also satisfied.