Math 116 Problem Set #6 Due Fri., Oct. 23, 2009, in class.

Read Apostol, Chapter 12, sections 12-14.

1. From Apostol, 12.11, pages 460-462: do problems 17(a,b) and 18.

2. From Apostol, 12.15, pages 467-468: do problems 1, 6, 7, 17.

3. a) Show that if we define a new product on $\mathbb{R}^2$ by

$$(a_1, a_2) \cdot (b_1, b_2) = 2a_1b_1 + a_1b_2 + a_2b_1 + a_2b_2,$$

then the usual laws of inner product (see Apostol, Theorem 12.2) are still satisfied.

b) In terms of coordinates, write an explicit formula for a new norm on $\mathbb{R}^2$ that is related to this inner product by the equation $\|v\|^2 = v \cdot v$.

c) Explain why the properties of norm given in Theorems 12.4 and 12.5 are automatically satisfied for this new norm (i.e. without the need to do any new computations).

4. In $\mathbb{R}^n$ (with $n \geq 2$), show that there is *no* way to define an inner product that gives the norm defined in problem 18 in Apostol, section 12.11. [Hint: Under this norm, what are the norms of e_1, e_2, and $e_1 \pm e_2$, where $e_1, \ldots, e_n$ are the unit coordinate vectors? What does this say about $e_1 \cdot e_2$ under such an inner product?]

5. Let V be as in problem 6 on Problem Set 5. Find all positive real numbers c such that the functions $f(x) = \cos cx$ and $g(x) = \sin cx$ are orthogonal.

6. Let V be as in problem 6 on Problem Set 5.

a) Explain why the Cauchy-Schwarz inequality (Theorem 12.3 of Apostol, page 452) holds for V and its associated inner product; and then explicitly state what it says for pairs of elements of V, in terms of integrals. (State both parts, including when the equality sign holds.) Then explicitly prove this assertion about integrals in the case that the two elements of V are the functions x^m and x^n, where m and n are positive integers.

b) Do the same for the Triangle Inequality (Theorem 12.5 of Apostol, page 454).