

*Reminder:* The second exam is on Monday, November 9 in class. One two-sided handwritten 5" x 7" index card is permitted during the exam, but no other aids.

Read Apostol, Chapter 14, sections 6-20.

1. From Apostol, 14.7, pages 524-525: do problems 3, 5, 7, 17; and from 14.9, pages 528-529: do problems 3, 5, 7.
2. From Apostol, 14.13, pages 535-536: do problems 1, 11, 13; from 14.15, pages 538-539: do problems 1 (do just for #3,5 of 14.9), 2, 7; and from 14.19, pages 543-545: 1, 2(a), 4, 8.
3. Let  $F : \mathbb{R} \rightarrow \mathbb{R}^n$  be a differentiable vector-valued function that parametrizes the motion of a particle in  $\mathbb{R}^n$  whose speed is always at most  $c$  (where  $c$  is some positive real number).
  - a) Prove that if  $a < b$  then  $\|F(b) - F(a)\| \leq c(b - a)$ . Also explain why this is reasonable from a geometric point of view.
  - b) Give an example of a function  $F$  and values  $a < b$  for which there is equality in part (a), and give another example in which there is a strict inequality.
4. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a twice differentiable function. Parametrize the plane curve  $y = f(x)$  by  $F(t) = (t, f(t))$ , and suppose that  $F(a) = (a, f(a))$  is an inflection point of this curve for some value of  $a$ . Prove that  $T'(a) = 0$ , where  $T(t)$  is the unit tangent vector to the curve at the point  $F(t)$ . Is the principal normal vector  $N(a)$  at  $F(a)$  defined?
5.
  - a) Find the arclength of the plane curve given parametrically by  $F(t) = (2t, \frac{t^3}{3} + \frac{1}{t})$ , for  $1 \leq t \leq 3$ .
  - b) Find the arclength of the plane curve whose graph is  $y = \log \cos x$  for  $0 \leq x \leq \pi/4$ . (Here  $\log$  is the natural logarithm.)
6. Consider the curve in  $\mathbb{R}^3$  given parametrically by  $F(t) = ti + t^2j + t^3k$ , where  $i, j, k$  are the unit basis vectors. Find the curvature at the origin, and find all points where the curvature is zero.