

Read Apostol, Chapter 8, sections 1-13.

1. From Apostol, 8.5, pages 311-312, do problems 1-6, 10.
 2. From Apostol, 8.7, pages 319-322, do problems 1, 2, 4, 8, 9, 18.
 3. From Apostol, 8.14, pages 328-329, do problems 3-6, 13, 14, 18.
 4. Let $a(x), b(x), c(x)$ be twice-differentiable functions $\mathbb{R} \rightarrow \mathbb{R}$.
 - a) Show that the set of solutions to the differential equation $y'' + a(x)y' + b(x)y = c(x)$ satisfies the axioms of a vector space (see §15.2) if and only if $c(x)$ is the constant function 0.
 - b) Show that if $a(x) = 0$, $b(x) = 1$, and $c(x) = 0$, then $\sin(x)$ and $\cos(x)$ are linearly independent elements of this vector space.
 5. Solve the initial value problem given by $y''' - y'' - 2y' = e^x$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 3$.
 6. a) Say a monetary quantity (e.g. an interest-bearing deposit, or the consumer price index) grows at a fixed rate of r percent per year, compounded continuously. Let C be the value at time $t = 0$. Write an initial value problem that corresponds to this situation, and solve this problem, obtaining a formula for this function of t in terms of r . Find the time t_0 that it takes for the quantity to double. What is the relationship between t_0 and r ? Give this both in precise form and numerically (with the value of any constant given to within .01).
 - b) In the situation of (a), suppose that at the end of each year, the quantity is i percent higher than at the beginning of that year. Find the relationship between i and r . Also give a precise expression for t_0 in terms of i . If $t_0 = k/i$, what is the numerical value of k (to within .01) if $i = 4$? 8? 16? How does k vary with i ? What is the limit of k as $i \rightarrow 0$? as $i \rightarrow \infty$?
- [Note: in part (a), r is the “interest rate before compounding”; in part (b), i is the “effective annual yield.”]