Math 116 Sample Exam #1 October, 2009.

Instructions: This exam consists of five problems. Do all five, showing your work and explaining your assertions. Give yourself 50 minutes. Each problem is worth 20 points.

1. Suppose that $T \subset S \subset \mathbb{R}$, and that $T \neq \varnothing$.
   a) Show that if S has a supremum, then so does T.
   b) Determine whether the converse also holds.

2. Let $f(x) = x^2[x]$, where $[x]$ is the greatest integer in x.
   a) On which closed intervals is f integrable?
   b) Evaluate $\int_0^2 f(x)dx$ if it exists.

3. a) In terms of ε, δ, write out explicitly the meaning of the statement that the function $f(x) = 3x$ is continuous at $x = 0$.
   b) Explicitly prove this assertion by finding an appropriate $\delta > 0$ for each $\varepsilon > 0$.

4. Let $f(x) = 4x - x^2$.
   a) Must the function f achieve a maximum value on every closed interval $[a, b]$? If so, explain why. If not, give a counterexample.
   b) Must the function f achieve a maximum value on every open interval (a, b)? If so, explain why. If not, give a counterexample.

5. Let f be a function with the property that $-x^2 \leq f(x) \leq x^2$ for all x.
   a) Determine whether f is necessarily continuous at $x = 0$. If so, explain why. If not, give a counterexample.
   b) Determine whether f is necessarily integrable on the closed interval $[-1, 1]$. If so, explain why. If not, give a counterexample.