Math 116 Sample Exam #2 November, 2009

Instructions: This exam consists of five problems. Do all five, showing your work and explaining your assertions. Give yourself 50 minutes. Each problem is worth 20 points.

1. a) Show that if $v \in \mathbb{R}^n$ has the property that $v \cdot w \geq 0$ for all $w \in \mathbb{R}^n$, then $v = 0$.
   b) Show that if $v, w \in \mathbb{R}^3$ and $v \times w = v - w$, then $v = w$.

2. a) Find a vector $z \in \mathbb{R}^3$ of length 1 that is orthogonal to the vectors $v = (1,1,0)$ and $w = (0,1,1)$.
   b) With v, w, z as in part (a), find the volume of the parallelepiped generated by these three vectors.

3. a) Find the focus and directrix of the parabola $y = 2x^2$.
   b) Find the equation of an ellipse that has foci at the points $(\pm 1, 0)$ and passes through the points $(0, \pm 1)$.

4. Let $P = (1,1,1)$ and $Q = (2,3,-1)$ in $\mathbb{R}^3$. Let L be the line in $\mathbb{R}^3$ that passes though P and Q.
   a) Find a differentiable function $F : \mathbb{R} \to \mathbb{R}^3$ that parametrizes the line L, such that $F(0) = P$, and such that the speed at every point is 1. Find the acceleration at each point.
   b) Find a plane in $\mathbb{R}^3$ that is perpendicular to L and passes through the point P.

5. Let $F : \mathbb{R} \to \mathbb{R}^2$ be a differentiable function, corresponding to the motion of a particle in the plane that traces out a curve C. Suppose that the speed is constant. For each of the following assertions, either give a proof or a counterexample.
   a) At every point of C, the acceleration vector is orthogonal to the velocity vector.
   b) At every point of C, the acceleration vector is a multiple of the position vector.