Math 314 Problem Set #9 Due week of March 25, 2019, in lab.

Reminder: Exam #2 will take place in class on Friday, March 29, on the material covered in class since the first exam up through Friday, March 22, and on homeworks from PS 6 through PS 9. Sample Exam 2 is available on the course web page. Those who submit the sample exam to their TA by 3pm on Wednesday, March 27, or who submit it in lecture on March 27, will get extra credit. There are also more study problems for Exam 2 on the course web page (as an aid to study, not for additional extra credit). During the actual exam, no electronic devices, notes, or other aids can be used, except for a single two-sided handwritten 5"x7" index card.

In Hoffman and Kunze, read Chapter 5 (all; Section 7 is optional).

1. From Hoffman and Kunze, Chapter 4, do: page 127, #5. From Chapter 5, do these problems: pages 148-150, #5; pages 155-156, #2; pages 162-163, #1 (first matrix only), 2(a), 3, 4.

2. Let $a(x), b(x) \in F[x]$, where F is a field. Let J be the set of all polynomials in $F[x]$ that are of the form $a(x)f(x) + b(x)g(x)$ with $f(x), g(x) \in F[x]$. Let $p(x)$ be the monic polynomial in J of smallest degree.

a) Show that $p(x)$ divides every polynomial in J, and in particular divides $a(x)$ and $b(x)$.

b) Show that *every* polynomial $q(x) \in F[x]$ that divides both $a(x)$ and $b(x)$ must also divide $p(x)$, and conclude that $p(x)$ has highest degree among all polynomials that divide both $a(x)$ and $b(x)$. [Hint: $p(x) \in J$.]

3. Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & c \end{pmatrix} \in M_n(F)$ where $c \in F$ is a scalar.

a) *Using row reduction*, determine for which $c \in F$ there is an inverse for A and find A^{-1} for each such c.

b) Compute the determinant of A.

c) *Using part* (b), determine for which $c \in F$ there is an inverse for A, and compute A^{-1} using the formula for inverses in terms of determinants. Does this agree with your answers to part (a)?

4. Let A be a 3×2 matrix and let B be a 2×3 matrix. Find $\det(AB)$. Explain your answer, and explain the connection to problem 4 on Problem Set #5. What are the possible values of $\det(BA)$?

5. Consider the system of linear equations $AX = B$, where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -3 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}.$$

Solve this system in three different ways:

a) By applying row reduction to the augmented matrix $(A \mid B)$.

b) By finding A^{-1} and writing $X = A^{-1}B$.

c) By Cramer's Rule.