Math 314 Problem Set #13 Due week of April 22, 2019, in lab.

Read Hoffman and Kunze, Chapter 6, Section 8; Chapter 7, Sections 1-3; and Chapter 8, Section 1.

1. From Hoffman and Kunze, do these problems: Chapter 6, pages 225-226, #1,3,6,15; Chapter 7, pages 230-231, #1,2,4; pages 241-244, #1,4(a); pages 249-251, #3.

2. From Hoffman and Kunze, Chapter 8, do these problems: pages 275-277, #1,3,4,9.

3. Find the rational canonical form of a 3×3 Jordan block.

4. Let V be a finite dimensional vector space over $\mathbb{R}$, let $T : V \to V$ be a linear transformation such that $T^r = I$ for some $r \geq 1$, and let $W \subset V$ be a T-invariant subspace.

a) Show that there is a projection map P of V onto W. [Hint: Pick a basis of W.]

b) For your map P, define $P' : V \to V$ by

$$P'(v) = \frac{1}{r} \sum_{i=0}^{r-1} T^{-i}(P(T^i(v))).$$

Show that P' is also a projection of V onto W. [Hint: What is the restriction of P' to W? What is the image of P'? Also explain why T is invertible and T^{-i} exists.]

c) Prove that for any $v \in V$, $P'(T(v)) = T(P'(v))$, and deduce that $\ker(P')$ is T-invariant.

d) Use part (c) to show that $V = W \oplus W'$ for some T-invariant subspace $W' \subset V$.

5. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Show that T *preserves length* (i.e. $\|T(x)\| = \|x\|$ for all $x \in \mathbb{R}^n$) if and only if T *preserves inner product* (i.e. $T(x) \cdot T(y) = x \cdot y$ for all $x, y \in \mathbb{R}^n$). [Hint: $\|x\|^2 = x \cdot x$, so $\|T(x+y)\|^2 = \cdots$.]